PROCYCLICAL PROMISES*

Jason Roderick Donaldson

Washington University in St. Louis

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Abstract

In this paper, I model a dynamic market economy in which debt contracts are subject to renegotiation. I find that a firm with more cyclical cash flows can borrow more than a firm with less cyclical cash flows, consistent with empirical evidence. The reason is that a firm with cyclical cash flows puts its creditors in a relatively strong bargaining position when debt is renegotiated. In general equilibrium, this leads to a collateral premium for assets that generate cyclical cash flows—they are expensive because they allow firms to borrow against them, scaling up their investments via leverage. Endogenous variation in enforcement frictions generates business cycle fluctuations in both firm-level and aggregate quantities consistent with stylized facts, even in the absence of capital accumulation or technology shocks.

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1 Introduction

Asset liquidation values are a key determinant of corporate debt capacity.[1] Williamson (1988) argues that this is because creditors are more willing to lend to a firm when they can liquidate its assets at high prices in the event of default. Shleifer and Vishny (1992) extend this argument. They point out that asset liquidation values depend on the state of the macroeconomy when liquidation actually occurs.[2] According to this argument, firms with less cyclical cash flows can borrow less than firms with more cyclical cash flows. The reason is that a firm with cyclical cash flows is relatively likely to default in a recession, when asset prices are depressed. Thus, creditors are relatively less willing to lend to a firm with cyclical cash flows, since they recover little when they liquidate its assets. However, there is limited empirical support for this prediction that debt capacity is decreasing in cash-flow cyclicality. In fact, Campbell, Polk, and Vuolteenaho (2010) and Maia (2010) find that, on the contrary, corporate leverage is increasing in cash-flow cyclicality.[3] Why?

In this paper, I build a general equilibrium model to address this question. I argue that even though firms with more cyclical cash flows repay less when asset liquidation occurs, they may repay more when asset liquidation does not occur. This is because asset liquidation values play an important role in determining the outcome of debt renegotiation, because just the *threat* of asset liquidation determines how much a firm ends up repaying its creditors when debt is renegotiated.[4] The model offers an explanation for why corporate leverage is positively related to cash-flow cyclicality and casts light on numerous other empirical facts about corporate policies and asset prices.[5]

[1]Empirical evidence that liquidation values are an important determinant of debt capacity is in Rampini and Viswanathan (2013), who find that the value of a firm's tangible assets is a key determinant of its debt capacity. Benmelech and Bergman (2009) show that collateral determines the interest rate charged on debt, using data on the redeployability of airline assets to overcome identification challenges.

[2]Empirical evidence for such macroeconomic effects is in Benmelech and Bergman (2011). That paper documents the importance of general equilibrium effects in asset markets for corporate borrowing, showing that a firm's bankruptcy increases the cost of debt for other firms in the same industry, since the liquidation of the bankrupt firm's assets depresses collateral values.

[3]In these papers, the measure of a firm's cash-flow cyclicality is the sensitivity of it is cash flows to aggregate output, or "cash-flow beta." I discuss this measure within the context of my model in Subsection 5.1. Note that cash-flow cyclicality is just one component of asset cyclicality, which is reflected in the fact that a firm's asset beta can be decomposed into its cash-flow beta and its discount-rate beta. I discuss this further in Subsection 5.2.

[4]Empirical evidence that liquidation values are important not only in the event of bankruptcy, but also in the event of renegotiation is in Benmelech and Bergman (2008). That paper shows that the liquidation values of assets are a key determinant of the outcome of renegotiation. Further, just the frequency of debt renegotiation affirms its importance. Using a sample of private loans to public firms, Roberts and Sufi (2009) find that over ninety percent of long-term debt contracts are renegotiated before maturity.

[5]I discuss the model's empirical content in Subsection 3.3 (following Proposition 4) and in Section 5.

Model preview. There is an infinite-horizon discreet-time environment in which overlapping generations of short-lived entrepreneurs borrow from long-lived investors. At each date, one of two equally likely states is realized, called a "boom" and a "recession." Asset prices are high in booms and low recessions.[6] There are two types of entrepreneurs, procyclical entrepreneurs who have high cash flows in aggregate booms and countercyclical entrepreneurs who have high cash flows in aggregate recessions.[7] Procyclical entrepreneurs make investments in a capital asset, called procyclical capital; countercyclical entrepreneurs make investments in a different capital asset, called countercyclical capital. In order to buy capital to make these investments, entrepreneurs borrow from investors. The amount that entrepreneurs can borrow is determined by the repayments investors expect to receive, and investors' ability to extract repayments is determined by the liquidation values of entrepreneurs' assets. This is because entrepreneurs may *divert* capital or *renegotiate* repayments, as in Hart and Moore (1998). Specifically, within each period the timing is as follows. First, an entrepreneur learns the cash flows of his project, which will accrue to him at the end of the period. Second, the entrepreneur either continues his project or diverts[8] a fraction $1-\theta$ of his capital and defaults. In the event of default, his creditor liquidates the remaining proportion θ of capital in the market and no cash flows accrue at the end of the period. Third, if the entrepreneur has not defaulted, the project's cash flows accrue. However, the entrepreneur renegotiates the repayment down to the liquidation value of capital.

Results preview. The first main result is that debt capacity is increasing in cyclicality, i.e. procyclical entrepreneurs can borrow more than countercyclical entrepreneurs. To see the mechanism behind this result, suppose that you are an investor lending to a countercyclical entrepreneur, namely an entrepreneur who has low cash flows in booms and high cash flows in recessions. How much will you be repaid in each state, i.e. in the boom and in the recession? In the boom, the countercyclical entrepreneur's cash flows are low. Thus, he has

[6]To preview the model, I take it has given that asset prices are higher in booms than in recessions. However, in the formal model, asset prices are determined in equilibrium and this is endogenous. Further, which of the two aggregate states is a boom and which is a recession is also endogenous, in so far aggregate productivity, output, and asset prices are higher in one state, the "boom," than in the other, the "recession." Moreover, these differences between the states are the result only of contracting frictions in equilibrium. This is formalized in Proposition 4.

[7]The results depend only on the assumption that one type of entrepreneur has more cyclical cash flows than the other type of entrepreneur; in particular, the extreme assumption that one entrepreneur has a countercyclical technology is not necessary.

[8]I focus on capital diversion for concreteness. However, there are several other micro-foundations that give nearly identical results; for example, if liquidation destroys a fraction $1-\theta$ of capital as in Shleifer and Vishny (1992) or if entrepreneurs must pay maintenance costs to prevent capital from depreciating as in Lorenzoni (2008). I choose diversion because it is tractable in this setting and has precedent in the finance literature, e.g. DeMarzo and Fishman (2007b) and DeMarzo and Sannikov (2006). Cf. footnote 12.

Entrepreneurs' Repayments

state	countercyclical entrepreneur	procyclical entrepreneur
boom	diverts and defaults repayment = $\theta \times$ asset value$_H$	continues and renegotiates repayment = asset value$_H$
recession	continues and renegotiates repayment = asset value$_L$	diverts and defaults repayment = $\theta \times$ asset value$_L$

Figure 1: The table above represents the actions (diversion or continuation) and repayments of procyclical entrepreneurs and countercyclical entrepreneurs. The subscripts H ("high") and L ("low") indicate the asset values are relatively high in booms and relatively low in recessions.

little incentive to continue his project and prefers to divert capital and default. As a result, you liquidate his assets; you receive a fraction θ of the entrepreneur's total asset value. In the recession, in contrast, the countercyclical entrepreneur's cash flows are high. Thus, he has a strong incentive to continue his project. He keeps his assets in place, but renegotiates his repayment and repays you the liquidation value of his capital. In summary, if you lend to the countercyclical entrepreneur, you receive a fraction θ of the value of capital assets in the boom and the entire value of capital assets in the recession.

Now suppose, analogously, that you are a investor lending to a procyclical entrepreneur, namely an entrepreneur who has low cash flows in recessions and high cash flows in booms. How much will you be repaid in each state? The procyclical entrepreneur is the complement of the countercyclical entrepreneur: he diverts capital in the recession and continues his project in the boom. As a result, you receive a fraction θ of the value of capital assets in the recession and the entire value of capital assets in the boom. Figure 1 summarizes the state-dependent repayments of the countercyclical entrepreneur and the procyclical entrepreneur.

Which entrepreneur are you willing to lend more to? The answer is the procyclical entrepreneur, since he can commit to repay you more (on average). This is because asset prices are higher in booms than in recessions. Thus, the countercyclical entrepreneur diverts capital when asset prices are high and the procyclical entrepreneur diverts when asset prices are low. Both entrepreneurs decrease your ability to extract repayments by diverting capital, but the countercyclical entrepreneur decreases this ability by *more* than the procyclical entrepreneur does. Hence, the procyclical entrepreneur can better commit to make his promised repayment—procyclical promises are credible promises. Therefore, you are willing to lend more to the procyclical entrepreneur than to the countercyclical entrepreneur: debt capacity is increasing in cyclicality.

The second main result is that is procyclical capital is more expensive than countercyclical capital—procyclical capital trades with a collateral premium. This is because it allows entrepreneurs to lever up, by the first main result above. Because procyclical entrepreneurs can borrow more than countercyclical entrepreneurs can, they can also buy more capital. Hence, the demand for procyclical capital is high relative to the demand for countercyclical capital, and so is the price.

In the model, capital prices and marginal productivity are high in booms relative to recessions, even though entrepreneurs' and investors' investment technologies do not change over time. This is because in booms entrepreneurs can lever up more, improving the *allocation* of capital. These changes in the capital allocation generate procyclical time-series fluctuations in firm cash flows, leverage, asset values, and investment, even without any capital accumulation or variation in corporate productivity. In my environment, all of these fluctuations result only from contracting frictions. They are not present in the Arrow–Debreu benchmark, in which entrepreneurs' asset holdings, leverage, and investment are constant in the time series. There is also no cross-sectional variation in this benchmark, i.e. procyclical entrepreneurs and countercyclical entrepreneurs borrow and invest the same amount and procyclical capital assets and countercyclical capital assets have the same price.

In addition to the Arrow–Debreu benchmark, I consider three benchmarks with varying degrees of enforcement frictions. The analysis of these benchmarks emphasizes that all of my main results rely on the interaction between the diversion friction and the renegotiation friction. Specifically, I consider benchmark models with diversion but not renegotiation, with renegotiation but not diversion, and with no borrowing whatsoever. In each of these benchmarks, there is no cross-sectional variation, in the sense that debt capacity and asset prices do not depend on cyclicality.

Finally, I discuss policy within my model. I ask whether there is a tax-subsidy scheme that could increase welfare. I find that taxing countercyclical entrepreneurs to subsidize procyclical entrepreneurs increases output. This is the case even though procyclical entrepreneurs and countercyclical entrepreneurs have equally productive investment technologies—the cost of the countercyclical entrepreneur investing one unit less due to the tax is equal to the benefit of the procyclical entrepreneur investing one unit more due to the subsidy. The reason that the tax-subsidy scheme increases output it that procyclical entrepreneurs can stretch their subsidies with leverage, given that debt capacity is increasing in cyclicality. Thus, a tax that decreases countercyclical entrepreneurs' investment by one unit generates a subsidy that increases procyclical entrepreneurs' investment by *more* than one unit.

Layout. The remainder of the Introduction contains a discussion of the literature. Section 2 presents the model. Section 3 presents the main analysis. It includes the results that debt capacity is increasing in cyclicality and that procyclical capital trades with a collateral premium. The Arrow–Debreu benchmark, the results about time-series fluctuations, and the analysis of the tax-subsidy scheme are also here. Section 4 presents four more benchmarks—the model with diversion but not renegotiation, the model with renegotiation but not diversion, and the model with no lending whatsoever as well as an explicit comparison of my model with Hart and Moore (1998). The analysis of these benchmarks underscores that the main results all stem from the interaction of the diversion and renegotiation friction in the full model. In Section 5, I discuss the model's empirical content. Section 6 concludes.

1.1 Related Literature

As discussed above, Shleifer and Vishny (1992) also use a model in which debt capacity is determined by asset liquidation values to study the question of how cash-flow cyclicality affects corporate leverage. They argue that debt capacity should be increasing in cash-flow cyclicality. My analysis suggests that allowing for debt renegotiation generates the reverse prediction: I find that corporate leverage is increasing in cash-flow beta, consistent with empirical evidence. Another paper that looks at the connection between corporate leverage and cash-flow beta is Ross (1985). He does not look at the effect of asset liquidation values, but rather focuses on the effects of corporate taxes in the presence of aggregate uncertainty. As in Shleifer and Vishny's (1992) theory, corporate leverage is increasing in cash-flow beta in Ross's (1985) model. His argument is that carried forward tax deductions are more valuable in recessions, when marginal utility is high, than in booms, when marginal utility is low. Thus, leverage is most valuable for firms that have high cash flows in recessions—those with low cash flow beta—since the present value of their tax deductions is highest. I do not include tax deductions in my model, but focus on how asset-liquidation values affect debt renegotiation.

Other papers focus on the cyclicality of a firm's assets (rather than cash flows) as a determinant of its leverage. In a model based on the trade-off theory, Choi (2013) predicts that corporate leverage is increasing in asset beta; he finds empirical support for this finding, as do Schwert and Strubalaev (2015). Asset beta can be decomposed as the sum of cash-flow beta and discount rate beta. My findings suggest that it is useful to distinguish between these two components of asset cyclicality in order to understand how cyclicality affects leverage. This is corroborated by Maia (2010), who finds that leverage is indeed increasing in cash-flow beta and decreasing in discount-rate beta.

My modeling framework builds on the literature that examines how capital liquidation values affect debt capacity in general equilibrium. Relevant papers in this literature include Alvarez and Jermann (2000), Geanakoplos (1997), Geanakoplos and Kübler (2004), Kehoe and Levine (1993), Kiyotaki and Moore (1997), and Lorenzoni (2008). The authors of these papers often focus on the effects on aggregate fluctuations in the time-series, which I also discuss in the context of my model. However, to the best of my knowledge, none of these papers studies my main focus, i.e. how cyclicality affects debt capacity and asset prices in the cross-section.

In my paper, aggregate productivity is procyclical because capital is better allocated in good times than in bad times. This is consistent with Eisfeldt and Rampini (2006) who find that capital reallocation is procyclical. They rationalize their findings in a model in which capital reallocation is costly and suggest that "the cost of reallocation needs to be substantially countercyclical" to be consistent with the data (p. 369). In my model, these countercyclical costs of capital reallocation are endogenous in so far as capital reallocation is endogenously more constrained in bad times than in good times. Rampini and Viswanathan (2010) also present a limited-enforcement financial contracting model in which capital is less efficiently deployed in downturns. They show how risk management failures prevent firms from seizing investment opportunities. I offer a complementary explanation based on the aggregate dynamics of the liquidation value of capital.

My paper is also related to the literature on dynamic financial contracting with limited enforcement, e.g. Albuquerque and Hopenhayn (2004), Bolton and Scharfstein (1990), DeMarzo and Fishman (2007a, 2007b), DeMarzo and Sannikov (2006), Gromb (1999), Hart and Moore (1989, 1994). Unlike this literature, I introduce dynamics via overlapping generations of entrepreneurs. This simplifies some aspects of the analysis, allowing me to study the interaction between the contracting frictions and endogenous capital prices analytically. Another paper that examines contractual dynamics in an OLG setup is Biais and Landier (2015). Unlike me, however, they do not study the interaction of this contracting problem with the equilibrium in a larger economy. One other paper that does examine this interaction is Cooley, Marimon, and Quadrini (2004). However, they do not include capital assets with different degrees of cyclicality, so they do not address the questions I take up in this paper.

In common with much of the work on liquidation values and debt capacity, I appeal to the theory of incomplete contracts to model collateral; the specific setup of the entrepreneurs' borrowing problem in my model builds on Hart and Moore (1998), as I discuss in detail in Subsection 4.4. A related strand of the literature studies how countercyclical agency costs, rather than collateral constraints, can amplify the business cycle, notably the "financial accelerator" literature following Bernanke and Gertler (1989). Rampini (2004) provides a related theory based on the idea that entrepreneurial risk-taking decreases in downturns, leading to a countercyclical inefficiency. The costs of such frictions are endogenously countercyclical in my model as well. Further, in equilibrium, these frictions *cause* the business cycle, in the sense that no aggregate fluctuations are present in the first-best case, in which these frictions are absent.

My paper is also related to the literature on how financial frictions lead to procyclical investment, such as Adrian and Shin (2014) and Danielsson, Shin, and Zigrand (2004, 2012). In these papers, credit supply is determined by frictions affecting lenders, whereas in my paper credit supply is determined by frictions affecting borrowers.

2 Model

In this section, I present the model. It is an infinite horizon discrete-time model in which long-lived investors lend to overlapping generations of short-lived entrepreneurs. There are two types of entrepreneurs, α and β. In equilibrium, α-entrepreneurs correspond to procyclical entrepreneurs and β-entrepreneurs correspond to countercyclical entrepreneurs, in the sense that α-entrepreneurs have high cash flows in booms and β-entrepreneurs have high cash flows in recessions. However, I do not use the labels "procyclical" and "countercyclical" in the formal model description, since booms and recessions are endogenous in the model, and thus so is cyclicality.

The main friction in the model is the limited enforceability of debt contracts. Entrepreneurs borrow to scale up their projects, but limit enforcement restricts their debt capacity. The specific enforcement frictions are capital diversion and renegotiation.

2.1 Environment, Goods, Players, and Technologies

Time is discrete and infinite. At each date $t \in \{..., -1, 0, 1, ...\}$, one of two equally-likely states is realized, denoted by $s_t \in \{a, b\}$. The process $\{s_t\}_t$ is i.i.d. I restrict attention to Markov allocations, so variables are indexed by the state s_t, rather than by the date t.

There are three goods, a consumption good called fruit, which serves as the numeraire, and two capital goods (or "assets"), called α-capital and β-capital. Both capital goods are perfectly durable are in constant supply K. Capital serves to collateralize loans in addition to producing fruit. $p^\tau_{s_t}$ denotes the price of capital good $\tau \in \{\alpha, \beta\}$ in state s_t.

There are three types of risk-neutral players: investors and two types of entrepreneurs. Each player is indexed by a technology $\tau \in \{\alpha, \beta, \gamma\}$. All technologies use capital goods to produce fruit. Investors operate technology γ, which uses either capital good. α-entrepreneurs operate technology α, which uses only α-capital, and β-entrepreneurs operate technology β, which uses only β-capital. k^τ_s denotes the quantity of τ-capital that a τ-entrepreneur holds in state s (he holds none of the other capital good, since he cannot use it to produce). At each date t, a unit of α-entrepreneurs and a unit of β-entrepreneurs are born. Entrepreneurs live for two dates. When they are young they borrow and invest; when they are old they produce and consume. Each entrepreneur is born with endowment w_{s_t} in state s_t. Entrepreneurs' endowments are greater in state a than in state b, $w_a > w_b = 0$.[9] This is the only exogenous difference between state a and state b. The entrepreneurs' technologies are productive but risky. α pays off only in state a and β pays off only in state b. Specifically, for investment k of α-capital at date t, α produces fruit according to

$$\alpha(k)(s_{t+1}) = \begin{cases} 2Ak & \text{if } s_{t+1} = a, \\ 0 & \text{if } s_{t+1} = b, \end{cases} \tag{1}$$

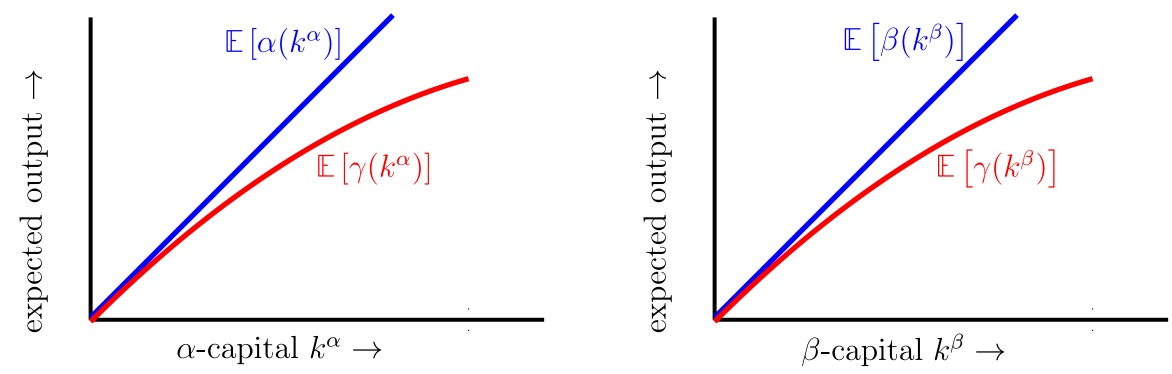
and, for investment k of β-capital at date t, β produces fruit according to

$$\beta(k)(s_{t+1}) = \begin{cases} 0 & \text{if } s_{t+1} = a, \\ 2Ak & \text{if } s_{t+1} = b. \end{cases} \tag{2}$$

(Entrepreneurs also continue to hold their capital k.) Note that since each state a or b is realized with probability half at date $t+1$, both types of entrepreneurs have marginal

[9] I make use of the "normalization" $w_b = 0$ only in the proof of Proposition 5.

Production Functions' Expected Output



(a) The expected output of the α- and γ-technologies from investment in α-capital.

(b) The expected output of the β- and γ-technologies from investment in β-capital.

Figure 2: The panels above illustrate that the α-technology and the β-technology both have expected marginal productivity A. The only difference between them is the state in which they pay off. Further, both the α-technology and the β-technology are better on average that the γ-technology.

productivity A:

$$\mathbb{E}\left[\,\alpha'(k)\,\right] = \mathbb{E}\left[\,\beta'(k)\,\right] = A, \tag{3}$$

as depicted in Figure 2. I refer to an entrepreneur's project as "successful" if it has a non-zero payoff. Thus, α-entrepreneurs are successful in state a and β-entrepreneurs are successful in state b. The table in Figure 3 summarizes old entrepreneurs' payoffs and young endowments.

Investors' technology γ employs either α-capital or β-capital, but not both. Each type of capital is operated by a unit of investors. In contrast to entrepreneurs, investors are long-lived with safe but relatively unproductive technologies. They consume at each date and discount the future with discount factor $1/R$. Their production technology γ is deterministic with decreasing returns-to-scale, $\gamma' > 0$ and $\gamma'' < 0$. γ is at most as productive as entrepreneurs' technologies, $\gamma'(0) = A$. Investors have deep pockets in fruit at each date. Since each capital good is in constant supply K, the quantity of τ-capital that investors hold is the quantity not held by τ-entrepreneurs, or $K - k_s^\tau$ in state s.[10]

[10] To economize on notation, I do not introduce a specific symbol for the quantity of capital held by investors. Specifically, rather than writing capital holdings in state s as $k_s^{\tau,\tau'}$, indexed by the type of capital $\tau \in \{\alpha, \beta\}$ and the type of player $\tau' \in \{\alpha, \beta, \gamma\}$, I write simply $k_s^\alpha = k_s^{\alpha,\alpha}$, $k_s^\beta = k_s^{\beta,\beta}$, and $K - k_s^\tau = k_s^{\tau,\gamma}$. This makes use of the facts that $k_s^{\alpha,\beta} = k_s^{\beta,\alpha} = 0$ and that, by market clearing, $k_s^{\tau,\alpha} + k_s^{\tau,\beta} + k_s^{\tau,\gamma} = K$.

Entrepreneurs' Endowments and Payoffs

	state $s_t = a$	state $s_t = b$
young α-entrepreneurs' endowment	w_a	w_b
young β-entrepreneurs' endowment	w_a	w_b
old α-entrepreneurs' payoff	$2Ak^{\alpha}_{s_{t-1}}$	0
old β-entrepreneurs' payoff	0	$2Ak^{\beta}_{s_{t-1}}$

Figure 3: The table above summarizes the entrepreneurs' endowments and payoffs.

2.2 Borrowing Contracts and Limited Enforcement

Entrepreneurs can borrow fruit bilaterally from investors via one-period contracts.[11] Thus, a contract in state s is a loan ℓ and promised repayment transfers $T(s)$ for each state $s \in \{a, b\}$ realized at date $t+1$. The enforcement of contracts is limited, because entrepreneurs can divert capital[12] and renegotiate repayments.[13] An entrepreneur can divert a fraction $1-\theta$ of his capital. If he diverts this capital, he destroys any fruit output of his project and he reneges on his debt; his creditor receives no repayment, but captures the proportion θ of capital that the entrepreneur cannot divert. I refer to this number θ, representing the proportion of assets that a creditor can seize, as the *enforceability* in the economy. If an entrepreneur does not divert capital, he renegotiates the repayment with this creditor. I assume that the entrepreneur renegotiates the repayment down to the creditor's seizure value of capital.[14] This contracting setup is built on Hart and Moore (1998).[15] See Figure 4 for a timeline representation of an entrepreneur's diversion and renegotiation decisions.

Without loss of generality, I restrict attention to contracts that do not induce diversion or renegotiation on the equilibrium path. Thus, the contractual repayments are subject to two constraints, a diversion constraint and a renegotiation constraint. The diversion constraint ensures that an entrepreneur with capital k prefers to continue his project and repay his debt than to divert a proportion $1-\theta$ of his capital, destroying his project's output but avoiding repayment. Specifically, if he continues his project he receives his cash flow $\tau(k)(s)$ plus holds capital worth $p_s^\tau k$, but he has to repay his debt $T(s)$. If he diverts, in contrast, he receives only the value of the proportion $1-\theta$ of this capital; he gets $(1-\theta)p_s^\tau k$. Thus,

[11] It is without loss of generality to restrict attention to (i) contracts with one-period maturity and (ii) contracts denominated in fruit. The reasons are as follows. (i) Entrepreneurs live for only two dates, so no long-term contracts are feasible. (ii) Fruit is the numeraire, which entrepreneurs use to buy capital the market. Allowing entrepreneurs to borrow in capital directly just amounts to a change of numeraire, which does not affect any of the results.

[12] As is common in the financial contracting literature, I model diversion as an entrepreneur literally stealing from its creditors. This extreme form of diversion is a real problem for firms. For example, Mironov (2013) calculates that Russian companies syphoned off upward of ten percent of GDP in both 2003 and 2004 and Akerlof and Romer (1993) describe related problems of corporate "looting" in the US. However, capital diversion also serves as a metaphor for more general incentive problems—DeMarzo and Fishman (2007a) show formally that a broad class of agency frictions can be modeled as cash-flow diversion.

[13] Empirical evidence on the real-world importance of debt renegotiation is in footnote 4.

[14] This is tantamount to the entrepreneur making his creditor a take-it-or-leave-it offer or having the bargaining power in renegotiation. The results below are not sensitive to this assumption, but rather are robust to general division of surplus between the entrepreneur and his creditor.

[15] I discuss the relationship between my model and Hart and Moore (1998) in more detail in Subsection 4.4.

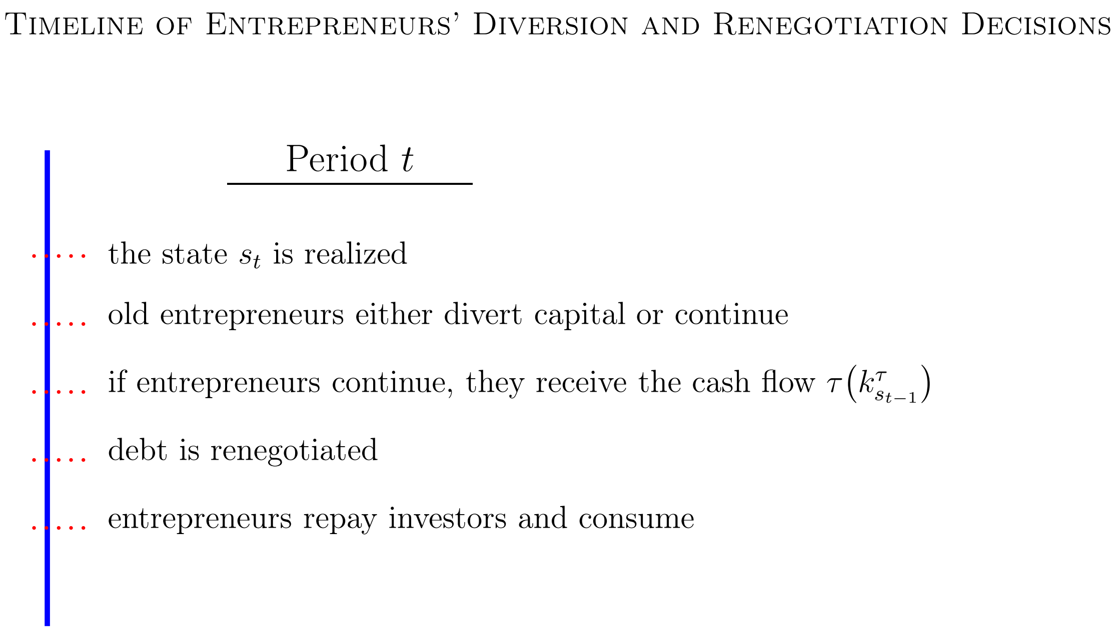


Figure 4: The timeline above represents a sequence of moves the give rise to the diversion constraint (equation (DC)) and renegotiation constraint (equation (RC)).

the diversion constraint says that in state s at date $t+1$ it must be that

$$\tau(k)(s) + p^{\tau}_s k - T(s) \geq (1-\theta)p^{\tau}_s k \tag{4}$$

or

$$T(s) \leq \tau(k)(s) + \theta p^{\tau}_s k. \tag{DC}$$

The renegotiation constraint ensures that an entrepreneur never pays his creditor more than the liquidation value of capital or that in state s at date $t+1$,

$$T(s) \leq p^{\tau}_s k. \tag{RC}$$

As discussed in Subsection 1.1, diversion and renegotiation are first-order real-world frictions. Their interaction generates the main results in this paper. I would like to emphasize that the way they interact in the model relies on the assumption that when the entrepreneur diverts capital, he destroys future output. Intuitively, if the entrepreneur is a farmer and his project is a fruit tree, then "diverting capital" corresponds to chopping down his tree and selling it for firewood. By liquidating his tree, he loses the fruit harvest. More generally, the assumption that diversion destroys capital gives the entrepreneur the incentive to continue his project when future cash flows are high.

2.3 Equilibrium

An equilibrium constitutes a capital allocation k_s^τ and a lending contract $\big(\ell_s^\tau, \big(T_s^\tau(a), T_s^\tau(b)\big)\big)$ for each type of entrepreneur $\tau \in \{\alpha, \beta\}$ in each state $s \in \{a, b\}$. The allocations maximize the entrepreneurs' expected consumption subject to their diversion constraints (DC), their renegotiation constraints (RC), and their budget constraints

$$w_s + \ell_s^\tau = p_s k_s^\tau \tag{BC}$$

at every date.

I focus on equilibria that are "interior" in the sense that investors are marginal in every market.[16] Thus, at the margin investors must be indifferent among lending to α-entrepreneurs, lending to β-entrepreneurs, consuming fruit, and buying capital to invest in their own technology. In other words, the reciprocal of investors' discount factor R equals their marginal expected return from lending and equals their marginal expected return from investing for each state s_t at each date t,

$$R = \frac{\mathbb{E}\left[T^\tau(s_{t+1}) \mid s_t\right]}{\ell_{s_t}^\tau} = \frac{\gamma'\big(K - k_{s_t}^\tau\big) + \mathbb{E}\left[p_{s_{t+1}}^\tau \mid s_t\right]}{p_{s_t}^\tau}. \tag{MC}$$

2.4 Definitions and Notations

In this section, I introduce several quantities and notations that are useful in the solution of the model. These are the average price of τ-capital $\bar{p}^\tau$, the one-period expected return from holding τ-capital ρ_t^τ, and the cyclicality of a τ-entrepreneur χ_t^τ.

The unconditional average price of capital $(p_a^\tau + p_b^\tau)/2$ comes up frequently below. Hence, I denote it with the shorthand $\bar{p}^\tau$.

Definition 1. *$\bar{p}^\tau$ denotes the <u>average price of capital</u>*

$$\bar{p}^\tau := \frac{p_a^\tau + p_b^\tau}{2}. \tag{5}$$

Since the distribution of states is stationary, the Markov allocation is also stationary, and,

[16] In an earlier version of this paper, I adopt a more primitive definition of equilibrium. I use the facts that investors are deep-pocketed and risk-neutral to establish that investors must be marginal in equilibrium; hence equation (MC) expresses necessary conditions for equilibrium. Since that analysis was unrelated to the main results, I impose the condition directly in the definition to streamline the analysis here.

thus, the conditional expected price equals the average price,

$$\mathbb{E}\left[p^{\tau}_{s_{t'}} \mid s_t\right] \equiv \bar{p}^{\tau} \tag{6}$$

for all t, $t' > t$.

The expected rate of capital gains from holding capital for one period is also useful. It is denoted by ρ^{τ}_{s}.

Definition 2. $\rho^{\tau}_{s_t}$ *denotes the expected one-period return on τ-capital (net of fruit output) in state s_t at date t,*

$$\rho_{s_t} := \frac{\mathbb{E}\left[p_{s_{t+1}} \mid s_t\right]}{p_{s_t}} \equiv \frac{\bar{p}}{p_{s_t}}. \tag{7}$$

I denote the aggregate productivity of capital by TFP. This concept comes up in the analysis when I study fluctuations in productivity over the business cycle. It is also useful to compare aggregate productivity in the baseline model and benchmark models; this demonstrates the connection between enforcement frictions and aggregate fluctuations. Since technologies are heterogeneous in this economy, aggregate productivity depends on the distribution of capital holdings among players.

Definition 3. *The aggregate productivity of capital in state s_t at date t is the expected average rate at which capital produces fruit at date $t+1$,*

$$\mathrm{TFP}_{s_t} := \frac{\text{expected output}}{\text{capital supply}} \tag{8}$$

$$\equiv \frac{\mathbb{E}\left[\alpha\big(k^{\alpha}_{s_t}\big) + \gamma\big(K - k^{\alpha}_{s_t}\big) + \beta\big(k^{\beta}_{s_t}\big) + \gamma\big(K - k^{\beta}_{s_t}\big)\right]}{2K}. \tag{9}$$

By this definition, aggregate productivity coincides with the total factor productivity calculated as the Solow residual from a linear aggregate production function—or, in the context of a neoclassical production function, from a constant-returns-to-scale Cobb–Douglas production function with capital share equal to one, which it is in this economy, since there is no labor.

Much of the analysis centers around the concept of cyclicality. In equilibrium, state a is a boom and state b is a recession, in the sense that aggregate output, aggregate productivity, and the prices of both capital goods are higher in state a than in state b. Thus, α-entrepreneurs are endogenously procyclical and β-entrepreneurs are endogenously countercyclical. However, before solving for the cyclicality of entrepreneurs in general equilibrium, it is useful to have a notion of cyclicality in partial equilibrium. One definition that emerges naturally from the analysis in partial equilibrium and coincides with cyclicality in general equilibrium is the co-movement of an entrepreneur's cash flows with the price of capital. (This concept is analogous to the "cash-flow beta" of a firm, in a way that I make precise in Subsection 5.1). In other words, an entrepreneur is procyclical if his cash flows are high when the market price of capital is high and countercyclical if his cash flows are high when the market price of capital is low. The next definition formalizes this notion.

Definition 4. χ^τ_t *denotes the* *cyclicality of τ-entrepreneurs in state s_t at date t*

$$\chi^\tau_{s_t} := \frac{\mathbb{P}\left[\tau \text{ is successful in } s_{t+1}\right] \mathbb{E}\left[p^\tau_{s_{t+1}} \,\middle|\, s_t\,,\, \tau \text{ is successful in } s_{t+1}\right]}{p^\tau_{s_t}}, \tag{10}$$

i.e., in state s,

$$\chi^\alpha_s := \frac{p^\alpha_a}{2p^\alpha_s} \quad \text{and} \quad \chi^\beta_s := \frac{p^\beta_b}{2p^\beta_s}, \tag{11}$$

since α-entrepreneurs succeed in state a and β-entrepreneurs succeed in state b.

2.5 Parameter Restrictions

I also make two restrictions on parameters.

Parameter Restriction 1. (Enforceability θ sufficiently large.)

$$\theta > 3 - 2R \tag{12}$$

This parameter restriction ensures that entrepreneurs do not always prefer to divert capital. Otherwise, if θ is small, an entrepreneur may prefer to divert capital even when it destroys a successful project. The reason that the discount rate R enters the expression is that it affects the price of capital, which in turn affects the payoff that the entrepreneur gets from diverting capital.

Parameter Restriction 2. (Entrepreneurs' wealth not too high.)

$$Rw_a < AK. \tag{13}$$

This parameter restriction ensures that the first-best outcome is not attained—i.e. that enforcement frictions bite. It says that entrepreneurs' wealth w_a cannot be too large relative to the total supply of capital K (remember that both capital goods have supply K).

3 Analysis

In this section, I derive the main results. First, I present the solution of the first-best model, i.e. the model without the enforcement frictions. In this benchmark, output, productivity, and capital prices are constant, both in the cross-section—across entrepreneurs and goods—and in the time series. Thus, the cross-sectional heterogeneity and aggregate fluctuations in the model are the result of enforcement frictions only. Next, I analyze the entrepreneurs' problems in partial equilibrium, taking capital prices as given. I find the first main result that more procyclical entrepreneurs have higher debt capacity. I proceed to analyze the model in general equilibrium. I first show that enforcement frictions generate macro fluctuations and that state a corresponds to a boom and state b corresponds to a recession and, thus, the α-technology is procyclical and the β-technology is countercyclical. Finally, I analyze capital prices in the cross-section. I find the second main result: α-capital trades at a price premium over β-capital. This is a collateral premium for procyclical capital, since procyclical capital mitigates enforcement frictions more effectively than countercyclical capital does.

3.1 First-best: The Arrow–Debreu Solution

In this section, I describe the equilibrium of the fricitonless economy, i.e. the equilibrium of the model without the diversion and renegotiation constraints in equations (DC) and (RC).

Proposition 1. (First-best equilibrium.) *In the economy without enforcement frictions, α-entrepreneurs hold all α-capital, $k_s^\alpha = K$, and β-entrepreneurs hold all β-capital, $k_s^\beta = K$. The equilibrium has the following properties.*

(i) Aggregate productivity is constant and equal to A, $\mathrm{TFP}_s = A$.

(ii) Aggregate output is constant and equal to $2AK$.

(iii) The prices of α-capital and β-capital are equal and constant; they equal the price of a perpetuity that pays coupon A,

$$p_s^\alpha = p_s^\beta = \frac{A}{R-1}. \tag{14}$$

The proposition summarizes the efficient allocation of capital. Entrepreneurs have the most productive technologies and thus hold all capital. They have constant productivity A (equation (3)), which coincides with the aggregate productivity because they hold all of the capital. In state a, α-entrepreneurs produce $2AK$ and, in state b, β-entrepreneurs produce $2AK$. Thus, there are no fluctuations in productivity or output in the time series. Capital prices are also constant, reflecting the marginal productivity A of capital.

The proposition above underscores that any cross-sectional variations of entrepreneurs' debt capacity and capital or time series fluctuations in productivity, output, and capital prices result only from limited enforcement.

To conclude this section, I note that the capital prices in the first-best model serve as an upper bound for the capital prices in the baseline model with enforcement fractions.

LEMMA 1. (PRICE BOUND.) *The price of capital in the baseline model is always less than that in the first-best,*

$$p_s^\tau \leq \frac{A}{R-1} \tag{15}$$

for $\tau \in \{\alpha, \beta\}$ *and* $s \in \{a, b\}$.

This result follows from the fact that the marginal productivity of investors is highest when entrepreneurs hold all the capital, and this margin productivity determines the price (equation MC).

3.2 Debt Capacity

In this section, I solve for entrepreneurs' debt capacity—the maximum quantity of fruit an entrepreneur can borrow from his creditor. The main result of this section is that an entrepreneur's debt capacity is increasing in his cyclicality χ. The reason is that procyclicality loosens the enforcement constraints.

To begin the analysis, I first establish which enforcement constraint binds in each state. This is the first step in connecting entrepreneurs' repayment constraints with their cyclicality. The diversion and renegotiation constraints are state-dependent—they depend on the output $\tau(k)(s)$ and the price of capital p_s^τ—but at most one constraint binds for an entrepreneur in each state. Since an entrepreneur produces no output when his project fails, he does not destroy any output by diverting capital. Thus, he always has incentive to divert in the event of failure. That is to say that the diversion constraint is always tighter than the renegotiation constraint in the failure state. In contrast, the cost of destroying capital is high in the success state. This gives an entrepreneur incentive not to divert capital. In fact, the diversion constraint is always slack in the success state (this follows from Parameter Restriction 1 and Lemma 1). Hence, the renegotiation constraint is always tighter than the diversion constraint in the success state. This is summarized in the next lemma.

LEMMA 2. (ENFORCEMENT CONSTRAINTS IN EACH STATE.) *An entrepreneur's diversion constraint is looser than his renegotiation constraint when he succeeds and his renegotiation constraint is looser than his diversion constraint when he fails.*

Thus, an α-entrepreneur's enforcement constraints are given by a renegotiation constraint in state a and a diversion constraint in state b: for an α-entrepreneur with capital k

$$T^{\alpha}(a) \leq p_a^{\alpha} k \tag{16}$$

and

$$T^{\alpha}(b) \leq \theta p_b^{\alpha} k. \tag{17}$$

Likewise, a β-entrepreneur's enforcement constraints are given by a renegotiation constraint in state b and a diversion constraint in state a: for a β-entrepreneur with capital k

$$T^{\beta}(b) \leq p_b^{\beta} k \tag{18}$$

and

$$T^{\beta}(a) \leq \theta p_a^{\beta} k. \tag{19}$$

This lemma illustrates the importance of the value of collateral even when entrepreneurs are not in distress. This is because the liquidation value of capital affects not only the diversion constraint but also the renegotiation constraint. Even though the diversion constraint is slack in the event of success, the maximum repayment is still limited by the renegotiation constraint. In fact, maximum repayments are even more sensitive to the value of capital in the success state than in the failure state, since in the failure state they appear with the discount θ due to capital diversion. This finding that renegotiation affects contracts mainly when entrepreneurs are not in distress is consistent with evidence in Roberts and Sufi (2009).

The lemma also provides an alternative interpretation of the diversion constraint. The lemma says that if an entrepreneur defaults, his creditor receives at most a fraction θ of the value of the entrepreneur's capital. This is equivalent to a model in which an entrepreneur defaults when his project fails and his creditor seizes his capital, but capital seizure is inefficient. Specifically, the creditor captures only a fraction θ of the entrepreneur's capital value in liquidation.

I now turn to the debt capacity $\ell^{\max}$ of an entrepreneur. This is the maximum amount that an entrepreneur can borrow subject to his creditor's being willing to lend. This means that $\ell^{\max}$ solves the program of maximizing the loan size ℓ subject to the renegotiation and diversion constraints in equations (DC) and (RC) as well as the entrepreneur's budget constraint in equation (BC) and the investor's participation constraint, captured by the condition in equation (MC).

Proposition 2. (Debt capacity.) *If an entrepreneur has cyclicality χ, then his debt capacity is given by*

$$\ell^{\max} = \frac{\theta\rho + (1-\theta)\chi}{R - \theta\rho - (1-\theta)\chi}\, w, \tag{20}$$

where ρ is the return on the capital good he invests in (see Definition 2).

Observe that the expression for an entrepreneur's debt capacity in equation (20) depends on the entrepreneur's technology only through its cyclicality χ. The other parameters in the expression are not specific to the entrepreneur. They are either primitives of the economy—as is the case for the enforceablity θ and discount rate R—or they are "macroeconomic variables"—as is the case with return on capital ρ. Further, it follows immediately from the expression in the proposition that $\ell^{\max}$ is increasing in χ—more procyclical entrepreneurs can borrow more. This is the main result of this section. It is illustrated in Figure 5 and restated in the next proposition.

Proposition 3. (Debt capacity is increasing in cyclicality.) *$\ell^{\max}$ is increasing in χ,*

$$\frac{\partial \ell^{\max}}{\partial \chi} = \frac{(1-\theta)Rw}{\big(R - \theta\rho - (1-\theta)\chi\big)^2} > 0. \tag{21}$$

This proposition says that, all else equal, a procyclical entrepreneur can borrow more than a countercyclical entrepreneur can. The reason is twofold. First, a procyclical entrepreneur succeeds when capital prices are high. Thus, his renegotiation constraint is relatively loose. Procyclicality effectively puts creditors in a strong bargaining position when entrepreneurs do not divert capital. Second, a procyclical entrepreneur's project fails when capital prices are low. Thus, the capital he diverts is relatively cheap. Hence, it is relatively less costly when procyclical entrepreneurs divert capital than when countercyclical entrepreneurs do. In other words, creditors can enforce repayment more effectively from procyclical entrepreneurs than from countercyclical entrepreneurs. Thus, procyclical entrepreneurs can keep more promises

Debt Capacity as a Function of Cyclicality

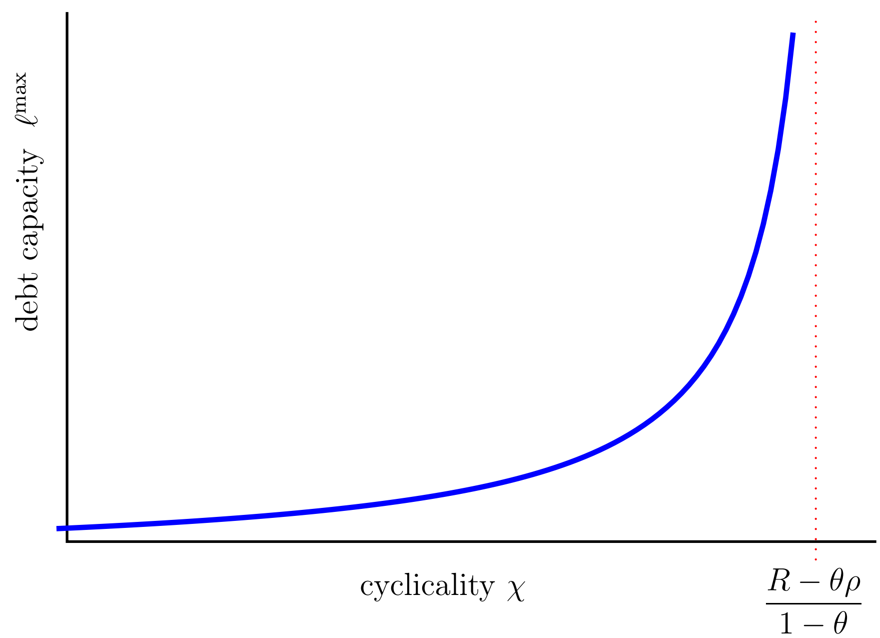


Figure 5: Debt capacity $\ell^{\max}$ is an increasing, convex function of cyclicality χ.

than countercyclical entrepreneurs—procyclical promises are credible promises. The ability to keep promises is the ability to borrow. Hence, procyclical entrepreneurs have more debt capacity than countercyclical entrepreneurs, as stated in the proposition.

I now proceed to explain the intuition behind the results in Proposition 2 and Proposition 3 in more detail. To do this, I give a heuristic derivation of the formula for debt capacity in Proposition 2. In this derivation, an entrepreneur pledges capital, borrows fruit, and buys more capital repeatedly at a single date; the process is analogous to tâtonnement in a Walrasian auction. Consider an entrepreneur with capital k_0 who wishes to borrow. He pledges this capital as collateral. This allows him credibly to promise to repay the total liquidation value of capital if his project succeeds and a proportion θ of the liquidation value of his capital if his project fails. I denote these repayments by $p_{\text{succ}}\,k_0$ and $\theta p_{\text{fail}}\,k_0$ in the success and failure states, respectively. These maximum repayments are tantamount to the binding diversion and renegotiation constraints as expressed in Lemma 2. The amount ℓ_1 he can borrow against his capital k_0 is equal to his discounted expected repayment, or, since success and failure are equally likely,

$$
\begin{aligned}
\ell_1 &= \frac{1}{R}\left(\frac{p_{\text{succ}}k_0}{2} + \frac{\theta p_{\text{fail}}k_0}{2}\right) && (22)\\
&= \frac{1}{R}\left(\theta\bar{p} + \frac{1}{2}(1-\theta)p_{\text{succ}}\right)k_0 && (23)\\
&= \left(\frac{\theta\rho + (1-\theta)\chi}{R}\right)pk_0 && (24)
\end{aligned}
$$

having divided by the price today p and substituted in for the definitions of ρ and χ from Definition 2 and Definition 4. This expression reveals that increasing cyclicality χ increases an entrepreneur's debt capacity. This is because the price of capital given success, p_{succ}, appears with a higher weight than the price of capital given failure, p_{fail}, since it is not discounted by θ due to capital diversion (this may be most transparent in the step from equation (22) to equation (23)).

The entrepreneur wishes to borrow to capacity. Thus, he uses the loan ℓ_1 to buy more capital $k_1 = \ell_1/p$. He then pledges this capital to borrow an amount ℓ_2 more fruit. He then repeats this process of pledging capital to borrow fruit, using the fruit to buy capital, and pledging the capital to borrow more fruit infinitely many times. Repeating the analysis above, we see that

$$
\begin{aligned}
\ell_2 &= \left(\frac{\theta\rho + (1-\theta)\chi}{R}\right)pk_1 && (25)\\
&= \left(\frac{\theta\rho + (1-\theta)\chi}{R}\right)^2 pk_0 && (26)
\end{aligned}
$$

and, more generally, after having borrowed n times, the entrepreneur takes another loan ℓ_n which is given by

$$
\ell_n = \left(\frac{\theta\rho + (1-\theta)\chi}{R}\right)^n pk_0. \tag{27}
$$

Hence, his debt capacity is given by the sum of al these additional loans,

$$
\begin{aligned}
\ell^{\max} &= \sum_{n=1}^{\infty}\ell_n && (28)\\
&= \sum_{n=1}^{\infty}\left(\frac{\theta\rho + (1-\theta)\chi}{R}\right)^n pk_0 && (29)\\
&= \frac{\theta\rho + (1-\theta)\chi}{R - \theta\rho - (1-\theta)\chi}\,pk_0, && (30)
\end{aligned}
$$

by the formula for the sum of a geometric series. Finally, substituting in for $pk_0 = w$ gives the expression as stated in Proposition 2.

I proceed to analyze the comparative statics of debt capacity, which are summarized in the next corollary.

COROLLARY 1. (DEBT CAPACITY COMPARATIVE STATICS.) *The debt capacity $\ell^{\max}$ has the following properties:*

(i) It is increasing in enforceability θ.

(ii) It is increasing in the return on capital ρ.

(iii) It is decreasing in the discount rate R.

(iv) Its sensitivity to cyclicality χ is decreasing in enforceability θ,

$$\frac{\partial^2 \ell^{max}}{\partial \theta \partial \chi} < 0. \tag{31}$$

I now comment briefly on the results enumerated in the corollary. (i) High enforceability limits the amount that entrepreneurs can steal, thereby increasing repayments to creditors. This makes creditors more willing to lend, i.e. it increases the debt capacity. Mathematically, this is a result of the fact that $\rho > \chi$, which follows immediately from the definitions in Subsection 2.4. (ii) When the return on capital is high, creditors have a high expected seizure value of collateral. This puts them in a strong bargaining position tomorrow, making them more willing to lend today. (iii) When the discount rate is high, creditors' value of repayments in the future is low. Thus, they are less willing to lend today for a given repayment. Keep in mind that, since players in the model are risk-neutral, the discount rate reflects only the time value of early consumption. In a more general model with risk-aversion, the discount rate would also be affected by the distribution of cash flows, since the marginal utility of consumption would be state-dependent. I discuss this further in Subsection 5.2. (iv) Cyclicality affects debt capacity less when enforceability is high. This is because procyclicality helps entrepreneurs to circumvent enforcement frictions and lever up. However, when enforceability is high, the value of circumventing these frictions is relatively low. This suggests a novel empirical implication of the model: the effect of cyclicality on corporate borrowing should be larger in environments in which enforceability is low, e.g. in countries that have weak property rights or industries that rely heavily on intangible capital (I discuss the empirical content of the model further in Section 5).

3.3 Time Series Fluctuations

In this section, I establish that state a is a "boom" and state b is a "recession" in the sense that aggregate output, aggregate productivity, and the prices of both capital goods are higher in state a than in state b. This is because entrepreneurs have higher endowments in state a than in state b, $w_a > w_b$. Recall that the differences between state a and state b are absent in the first best (Proposition 1). Thus, they result from enforcement frictions alone. To find the results about aggregate quantities, I proceed to characterize individual players' capital demands as well as the system of equations defining capital prices.

The first lemma says that entrepreneurs exhaust their debt capacities, so that their equilibrium borrowing equals the bound in Proposition 2.

Lemma 3. *Entrepreneurs borrow to capacity, $\ell_s^\tau = \ell^{\max}\left(\rho_s^\tau, \chi_s^\tau\right)$, as defined in Proposition 2.*

The intuition for this result is as follows. Entrepreneurs' technologies are at least as productive as investors'. Thus, entrepreneurs want to invest as much as they can in their projects. To maximize their capital investment, they borrow to capacity.

Now that I have solved for entrepreneurs' loan sizes ℓ_s^τ, I can write down their capital holdings as a function of prices directly from their budget constraints (equation (BC)). These capital holdings are summarized in the next corollary.

Corollary 2. (Capital holdings.) *In equilibrium, α-entrepreneurs hold α-capital*

$$k_a^\alpha = \frac{2Rw_a}{(2R-1)p_a^\alpha - \theta p_b^\alpha} \tag{32}$$

in state a and

$$k_b^\alpha = \frac{2Rw_b}{(2R-\theta)p_b^\alpha - p_a^\alpha} \tag{33}$$

in state b and β-entrepreneurs hold β-capital

$$k_a^\beta = \frac{2Rw_a}{(2R-\theta)p_a^\beta - p_b^\beta} \tag{34}$$

in state a and

$$k_b^\beta = \frac{2Rw_b}{(2R-1)p_b^\beta - \theta p_a^\beta} \tag{35}$$

in state b.

The expressions above reveal that entrepreneurs' capital holdings today are relatively more sensitive to capital prices in success states than to capital prices in failure states. This is because a procyclical entrepreneur—i.e. an entrepreneur who succeeds when capital prices are high—has high debt capacity (Proposition 2). Since he can borrow more than a countercyclical entrepreneur, he can also buy more capital. In other words, since ℓ_s^τ is increasing in cyclicality, so is k_s^τ.

The expressions above for the capital holdings, combined with the investors' marginal pricing equation (equation (MC)), give a system of equations that defines the capital prices.

LEMMA 4. (PRICING EQUATIONS.) *The prices of α- and β-capital solve the following system of equations: for each $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$,*

$$Rp_s^\tau = \bar{p}^\tau + \gamma' \left(K - k_s^\tau\right), \tag{36}$$

where k_s^τ is as given in Corollary 2.

The preliminary results above allow me to compare macroeconomic outcomes in state a and state b. These are summarized in following proposition, which is the main result of this section.

PROPOSITION 4. (AGGREGATE FLUCTUATIONS.) *State a is a "boom" and state b is a "recession" in the sense that the following statements hold.*

(i) Aggregate output is higher in state a than in state b.

(ii) Aggregate productivity is higher in state a than in state b, $\mathrm{TFP}_a > \mathrm{TFP}_b$.

(iii) The prices of both capital goods are higher in state a than in state b, $p_a^\alpha > p_b^\alpha$ and $p_a^\beta > p_b^\beta$.

This proposition describes the aggregate fluctuations between the two states in the model. These fluctuations exist even though individual productivity and the supply of investment capital do not change over time—all players have the same technologies at each date and there is no production or destruction of capital. All fluctuations correspond to changes in the *allocation* of capital—capital is more efficiently allocated in state a than in state b.

As I touched on in the Introduction, these dynamics are consistent with empirical evidence about fluctuations of both aggregate and firm-level variables. At the aggregate level, capital prices are procyclical, consistent with the data (IMF Research (2000)). In the model, capital prices are high in booms because their marginal productivity is high in booms. I.e. TFP is procyclical. This is a fact that Basu and Fernald (2001) refer to as an "essential feature of business cycles" (p. 225). This positive correlation between aggregate output and productivity occurs in my model even though all production technologies have (weakly) decreasing returns to scale. Thus, at the individual level, output and productivity are negatively correlated by definition. However, at the aggregate level they are positively correlated due to fluctuations in efficiency of capital allocation. This seeming contradiction is resolved by the cyclicality of capital allocation. Capital is reallocated to its efficient use in booms, and the value of capital that is reallocated is higher in booms, consistent with findings in Eisfeldt and Rampini (2006). Further, my model suggests that capital reallocation is endogenously more constrained in bad times than in good times, suggesting that the cost of reallocation is countercyclical. This idea finds support in Eisfeldt and Rampini's statement that "the cost of reallocation needs to be substantially countercyclical" to be consistent with the data (p. 369). Rampini and Viswanathan (2010) also present a limited-enforcement financial contracting model in which capital is less efficiently deployed in downturns. They show how risk management failures prevent firms from seizing investment opportunities. I offer a complementary explanation based on the aggregate dynamics of the liquidation value of capital.

The procyclicality of TFP reflects the fact that entrepreneurs borrow and invest more in state a than in state b. Corporate financing is indeed procyclical (Begenau and Salomao (2014)). Further, entrepreneurs' leverage is procyclical. This is consistent with Korteweg and Strebulaev's (2015) finding that target leverage is procyclical and with Korajczyk and Levy's (2003) finding that constrained firms' leverage is procyclical, given that all entrepreneurs in the model borrow to capacity—or hit their financial constraints (Lemma 3).[17] Finally, note that evidence on the procyclicality of corporate investment is in Dang and Wu (2015).

3.4 Capital Prices: The Collateral Premium

In this section, I turn to the prices of capital in the cross-section. The main result is that procyclical capital (α-capital) trades at a premium over countercyclical capital (β-capital).

[17] Halling, Yu, and Zechner (forthcoming) point out that the empirical findings in Korteweg and Strebulaev (2015) and Korajczyk and Levy (2003) capture only the direct effect of the business cycle on leverage, but do not take into account how leverage determinants change over the business cycle. The effect in my model reflects this direct effect. In my model, this effect reflects time-varying enforcement frictions.

This is a collateral premium, reflecting the fact that procyclical capital is the better collateral for loans.

Before I turn to the collateral premium, I derive an alternative expression for the average prices of the capital goods.

LEMMA 5. (AVERAGE CAPITAL PRICE AS PRICE OF A PERPETUITY.) *The price of τ-capital is the price of a perpetuity that pays a dividend equal to its average marginal productivity, i.e. a perpetuity with*

$$\text{dividend} = \frac{\gamma'(K - k_a^\tau) + \gamma'(K - k_b^\tau)}{2}, \tag{37}$$

so

$$\bar{p}^\tau = \frac{\gamma'(K - k_a^\tau) + \gamma'(K - k_b^\tau)}{2(R-1)}. \tag{38}$$

The lemma demonstrates that the price of a capital good is linearly related to its average productivity. Further, it provides the following dynamic intuition for the value of capital. Consider an investor holding capital for the long term. In state s, the marginal value of a unit of τ-capital is $\gamma'(K - k_s^\tau)$. Hence, the investor's value of capital at date t is the discounted sum of these "dividends":

$$\begin{aligned} \bar{p}^\tau &= \mathbb{E}_t\left[\gamma'(K - k_{s_t}^\tau)\right] + \frac{\mathbb{E}_t\left[\gamma'(K - k_{s_{t+1}}^\tau)\right]}{R} + \frac{\mathbb{E}_t\left[\gamma'(K - k_{s_{t+2}}^\tau)\right]}{R^2} + \cdots \qquad (39) \\ &= \frac{\gamma'(K - k_a^\tau) + \gamma'(K - k_b^\tau)}{2(R-1)}, \qquad (40) \end{aligned}$$

as given in the lemma, where I used the fact that the distribution of s_t is stationary and summed the geometric series.

I now state the main result of this section: that procyclical capital trades at a premium over countercyclical capital.

PROPOSITION 5. (THE COLLATERAL PREMIUM OF PROCYCLICAL CAPITAL.) *Procyclical capital trades at a premium over countercyclical capital, $p_a^\alpha > p_a^\beta$ and $p_b^\alpha > p_b^\beta$.*

The basic intuition for the result is that procyclical capital serves better as collateral than countercyclical capital, which increases the demand for procyclical capital, generating a price premium in equilibrium. In more detail, the mechanism behind the result works as follows. All entrepreneurs are constrained and scale up their projects to capacity. Procyclical entrepreneurs can scale up more, due to their higher debt capacity (Proposition 2). Thus, entrepreneurs hold relatively more procyclical capital than countercyclical capital. As a result, investors are left holding little procyclical capital relative to countercyclical capital. The investors' marginal productivity of procyclical capital is thus higher than their marginal productivity of countercyclical capital, since their production function γ is decreasing-returns-to-scale. This marginal productivity sets the price, resulting in a higher price of procyclical capital than countercyclical capital in equilibrium. In summary, prices are set by investors (the marginal players), but cross-sectional price differences are the result of the difference in the borrowing constraints of entrepreneurs (the infra-marginal players).

3.5 Policy: Taxes and Subsidies

In this section, I discuss one policy implication of the model. I find that taxing countercyclical entrepreneurs to subsidize procyclical entrepreneurs can improve welfare, even though it can amplify aggregate fluctuations and increase inequality among entrepreneurs.

The tax-subsidy scheme I consider is a tax that decreases a countercyclical entrepreneur's endowment w_s by ϵ and transfers it to a procyclical entrepreneur, increasing his endowment by ϵ. The next proposition states that this tax is welfare improving.

Proposition 6. (Tax on countercyclical entrepreneurs.) *A tax-subsidy scheme that transfers endowments from a countercyclical entrepreneur to a procyclical entrepreneur increases expected output, i.e. the entrepreneurs' expected output in state s,*

$$A\big(k_s^{\alpha}(w_s+\epsilon)+k_s^{\beta}(w_s-\epsilon)\big)=2RA\left(\frac{w_s+\epsilon}{2Rp_s^{\alpha}-p_a^{\alpha}-\theta p_b^{\alpha}}+\frac{w_s-\epsilon}{2Rp_s^{\beta}-p_b^{\beta}-\theta p_a^{\beta}}\right), \tag{41}$$

is increasing in ϵ. With appropriate ex post transfers, the scheme leads to a Pareto improvement.

The reason that taxing countercyclical entrepreneurs to subsidize procyclical entrepreneurs is beneficial in this economy is that procyclical entrepreneurs can stretch their endowments further via leverage. With an additional unit of fruit endowment, a procyclical entrepreneur can borrow and invest more than a procyclical entrepreneur can, since debt capacity is increasing in cyclicality (Proposition 2). This effect on investment implies that a transfer of a countercyclical entrepreneur's endowment to a procyclical entrepreneur allows entrepreneurs in aggregate to hold more capital. Since entrepreneurs are more productive than investors, this increases expected output in aggregate, which establishes the result.

The fact that this scheme in conjunction with ex post transfers can implement a Pareto improvement follows from risk-neutrality. Since expected output is higher in total, it can be reallocated to make the expected payoff higher for each player. This is a Pareto improvement when players are risk-neutral.

There is one caveat to the result. The analysis is done for a tax and subsidy from only one entrepreneur to another, not from all countercyclical entrepreneurs to all procyclical entrepreneurs. This simplifies the analysis, because in this case the intervention has no effect on capital prices, since each entrepreneur is small.

4 Benchmarks

In this section, I show that both the diversion constraint and the renegotiation constraint (equations (DC) and (RC)) are necessary for the main results. Without the interaction between these constraints, debt capacity does not depend on cyclicality and there is no collateral premium for procyclical capital, i.e. Proposition 2 and Proposition 5 do not hold in benchmark models in which either of the constraints is removed. I also show that the capital prices coincide if there are no credit markets.

4.1 Benchmark: Diversion without Renegotiation

In this section, I consider the benchmark without the renegotiation constraint (equation (RC)). In this case, a τ-entrepreneur's maximum repayment in both the success state and the failure state is given by his binding diversion constraint,

$$T(s) = \tau(k)(s) + \theta p_s k. \tag{42}$$

Thus, for a given capital holding k, the maximum an entrepreneur can borrow at date t (the analogy of equation (22)) is given by

$$\ell = \frac{\mathbb{E}_t\left[\tau(k)(s_{t+1}) + \theta p_{s_{t+1}} k\right]}{R} = \frac{A + \theta \bar{p} k}{R}. \tag{43}$$

This expression shows that, absent renegotiation, an entrepreneur's ability to borrow does not depend asymmetrically on the price of capital in the success and failure states; hence it does not depend on his cyclicality. The next lemma states the equilibrium debt capacity, which is independent of cyclicality.

PROPOSITION 7. *In the benchmark without renegotiation, debt capacity does not depend on cyclicality,*

$$\ell^{\max} = \frac{A + \theta\rho w}{R - \theta\rho}. \tag{44}$$

I now turn to equilibrium capital prices. Because debt capacity does not depend on cyclicality, the capital holdings of α-entrepreneurs equal the capital holdings of β-entrepreneurs in each state. Hence, the pricing equations for α- and β-capital coincide and there is no collateral premium for procyclical capital. I state this formally in the next proposition.

PROPOSITION 8. *In the benchmark without renegotiation, there is no collateral premium,* $p_a^\alpha = p_a^\beta$ *and* $p_b^\alpha = p_b^\beta$.

4.2 Benchmark: Renegotiation without Diversion

In this section, I consider the benchmark without the diversion constraint (equation (DC)). In this case, a τ-entrepreneur's maximum repayment in both the success state and the failure state is given by his binding renegotiation constraint,

$$T(s) = p_s k. \tag{45}$$

Thus, for a given capital holding k, the maximum an entrepreneur can borrow at date t is given by

$$\ell = \frac{\mathbb{E}_t\left[p_{s_{t+1}} k\right]}{R} = \frac{\theta \bar{p} k}{R}. \tag{46}$$

This is the analogy of equation (22) in the baseline and equation (43) in the benchmark without renegotiation. The expression shows that, absent diversion, an entrepreneur's ability to borrow does not depend asymmetrically on the price of capital in the success and failure states; hence it does not depend on his cyclicality. The next lemma states the equilibrium debt capacity, which is independent of cyclicality.

PROPOSITION 9. *In the benchmark without diversion, debt capacity does not depend on cyclicality,*

$$\ell^{\max} = \frac{\rho w}{R - \rho}. \tag{47}$$

I now turn to equilibrium capital prices. The logic is identical to that in the benchmark with diversion but no renegotiation in the previous section (Subsection 4.1): because debt capacity does not depend on cyclicality, the capital holdings of α-entrepreneurs equal the capital holdings of β-entrepreneurs in each state. Hence, the pricing equations for α- and β-capital coincide and there is no collateral premium for procyclical capital. I state this formally in the next proposition.

PROPOSITION 10. *In the benchmark without diversion, there is no collateral premium, $p_a^\alpha = p_a^\beta$ and $p_b^\alpha = p_b^\beta$.*

4.3 Benchmark: No Lending

In this section, I consider the benchmark with no lending whatsoever. In this case, entrepreneurs can invest only their endowments. Thus, in state s the demand for τ-capital is

$$k_s^\tau = \frac{w_s}{p_s^\tau} \tag{48}$$

for both capital goods, $\tau \in \{\alpha, \beta\}$. The expression implies that the pricing equations for α-capital and β-capital are identical. Hence, as in the other benchmark models above, there is no difference in the prices of procyclical capital and countercyclical capital. The next proposition states this formally.

PROPOSITION 11. *In the benchmark with no lending, there is no collateral premium, $p_a^\alpha = p_a^\beta$ and $p_b^\alpha = p_b^\beta$.*

4.4 Benchmark: Hart and Moore (1998)

In this subsection, I discuss a different kind of benchmark. Here, I recast the interaction between an entrepreneur and his investors as a variant of the model in Hart and Moore (1998), which serves as a benchmark against which I can compare my model above. This analysis also clarifies the micro-foundations of the diversion constraint (equation (DC)). Below, I first sketch a three-date model and derive the diversion constraint. I then explain how the timeline I present in Subsection 2.2 captures the frictions in Hart and Moore (1998) with one key difference: in my model an entrepreneur can divert all cash flows as well as a fraction $1-\theta$ of capital assets, whereas in Hart and Moore (1998) the entrepreneur can divert only cash flows.

Consider the following variant of Hart and Moore (1998). There are three dates, $t' \in \{0, 1, 2\}$. The state of nature s is realized at $t' = 1$. The entrepreneur borrows from an investor at $t' = 0$ and buys capital assets k. He promises to repay T_1 at $t' = 1$ and T_2 at $t' = 2$. At any point the entrepreneur can divert all his cashflow and a fraction $1 - \theta$ of his capital assets, so he will always divert at the final date. In state s, the entrepreneur's project generates cashflow $\tau(k)(s)$ at $t' = 2$. If the entrepreneur diverts at $t' = 1$, he defaults on all of his debt but foregoes these date-2 cashflows.

I now derive the diversion constraint at $t' = 1$, which is a constraint on the total repayment $T_1 + T_2$. The entrepreneur diverts at $t' = 1$ if his payoff from defaulting and forgoing his project exceeds his payoff from continuing and repaying his debt at $t' = 1$, or

$$\tau(k)(s) + p_s k - T_1 - T_2 \geq (1 - \theta) p_s k \tag{49}$$

or

$$T_1 + T_2 \leq \tau(k)(s) + p_s k. \tag{50}$$

This is exactly the diversion constraint in equation (DC). Note further that renegotiation in Hart and Moore (1998) yields the renegotiation constraint in equation (RC) immediately. Thus, the analysis of my model is equivalent to an OLG version of Hart and Moore (1998) with each period divided into sub-periods—at each date t there are two sub-periods $t' = 1$ and $t' = 2$—so old entrepreneurs repay their debt in two stages. In the baseline model, I simplify the analysis by assuming that an entrepreneur *first* decides whether to divert and *second* renegotiates his repayment if he has not diverted. In particular, at the time that the entrepreneur's debt matures, it is too late to divert. The analysis in this section demonstrates that simplifying the timeline in this way is without loss of generality.

5 Empirical Content and Risk Aversion

In this section, I focus on the empirical content of my model. I first present evidence consistent with the result that debt capacity is increasing in cyclicality (Proposition 3). I then discuss the prices of financial assets. To do this, I enrich the environment to include risk aversion. I suggest my model may cast light on some stylized facts in asset pricing. Finally, I discuss a number of other empirical predictions of my model. Some of these are new, and I describe how they may be tested.

5.1 Cyclicality, Cash-flow Beta, and Corporate Leverage

In this section, I argue that procyclical firms in my model are procyclical in the sense that they have high cash-flow beta, i.e. their cash flows are sensitive to the market's cash flows. Thus, my finding that debt capacity is increasing in cyclicality (Proposition 3) says roughly that corporate leverage is positively correlated with cash-flow beta. This is true in the data (Campbell, Polk, and Vuolteenaho (2010) and Maia (2010)).

I now compute the cash-flow betas of α- and β-entrepreneurs. Formally, I define an entrepreneur's cash-flow beta as the projection of his cash flow τ on the total market cash flows, which I denote here by Y,[18]

$$\text{beta}^{\tau}_{\text{CF}} := \frac{\text{Cov}\left[\,\tau\,,Y\,\right]}{\text{Var}\left[Y\right]}. \tag{51}$$

Since $\alpha(b) = 0$ and $\beta(a) = 0$, the covariance above simplifies to

$$\text{Cov}\left[\,\tau\,,Y\,\right] = \begin{cases} A(2Y_a - \bar{Y}) & \text{if } \tau = \alpha, \\ A(2Y_b - \bar{Y}) & \text{if } \tau = \beta, \end{cases} \tag{52}$$

where $\bar{Y} := (Y_a + Y_b)/2$. Since $Y_a > Y_b$ by Proposition 4, this implies that cash-flow beta is a measure of cyclicality in my model, in the sense that α-entrepreneurs have higher cash-flow beta than β-entrepreneurs.

5.2 Risk Aversion and Stock Prices

In this section, I discuss the implications of my model for pricing financial assets. So far, I have focused on corporate finance frictions and, thus, I have restricted attention to risk-neutral players. To study the effect of my corporate-finance mechanism on financial asset prices, I introduce the effect of risk-aversion in a reduced-form way. Specifically, I assume that there is an exogenous stochastic discount factor that adjusts for the decreasing marginal utility of a representative investor. Since output is higher in state a than in state b (by Proposition 4), the representative investor's consumption is higher in state a than in state b. Thus, the change of measure $d\mathbb{Q}/d\mathbb{P}$ from the physical measure $\mathbb{P}$ to the pricing measure

[18] Sometimes this is referred to "cash flow-cash flow beta," because it focuses both on firm cash flows and market cash flows (see Maia (2010) for a discussion). My result here is robust to other definitions of cash-flow beta.

$\mathbb{Q}$ increases the weight on state b and decreases the weight on state a,

$$\frac{d\mathbb{Q}}{d\mathbb{P}}(a) < 1 < \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{53}$$

Let S_s^τ denote the discounted present value of the output of (all future generations of) τ-entrepreneurs in state s, i.e. the price of a "stock" in τ-entrepreneurs. By the definition of the measure $\mathbb{Q}$, this price equals the discounted sum of expected future cash flows:

$$S_{s_t}^\tau = \mathbb{E}_t^{\mathbb{Q}}\left[\sum_{t'=1}^{\infty} \frac{\tau\left(k_{t'-1}^\tau\right)}{R^{t'}}\right] \tag{54}$$

$$= \mathbb{E}_t\left[\sum_{t'=1}^{\infty} \frac{d\mathbb{Q}}{d\mathbb{P}} \frac{\tau\left(k_{t'-1}^\tau\right)}{R^{t'}}\right]. \tag{55}$$

Making use of the simplifying assumption that $w_b = 0$, we have that entrepreneurs invest no capital in state b, $k_b^\alpha = k_b^\beta$, so output is positive at date t only if $s_{t-1} = a$. Thus, α- and β-entrepreneurs each succeed with probability 1/4—α-entrepreneurs succeed in state a following state a and β-entrepreneurs succeed in state b following state a. As a result, we can express the average stock prices $\bar{S}^\tau = (S_a^\tau + S_b^\tau)/2$ as geometric series as follows:

$$\bar{S}^\alpha = \sum_{t'=1}^{\infty} \frac{1}{4} \frac{d\mathbb{Q}}{d\mathbb{P}}(a) \frac{\tau\left(k_a^\alpha\right)}{R^{t'}} \tag{56}$$

$$= \frac{k_a^\alpha}{4(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(a) \tag{57}$$

and

$$\bar{S}^\beta = \sum_{t'=1}^{\infty} \frac{1}{4} \frac{d\mathbb{Q}}{d\mathbb{P}}(b) \frac{\tau\left(k_a^\beta\right)}{R^{t'}} \tag{58}$$

$$= \frac{k_a^\beta}{4(R-1)} \frac{d\mathbb{Q}}{d\mathbb{P}}(b). \tag{59}$$

Note that the price of the β-stock depends on the price of risk adjustment $d\mathbb{Q}/d\mathbb{P}$ in state b, but it still depends on the capital investment k_a^β in state a, since investments are made in the boom a.

Which stock is more expensive? Stock in a procyclical entrepreneur S^α or stock in a countercyclical entrepreneur S^β? To answer this, compute the difference between the average prices of the α- and the β-stock expressed above:

$$\bar{S}^{\alpha} - \bar{S}^{\beta} = \frac{1}{4(R-1)} \left(k_a^{\alpha} \frac{d\mathbb{Q}}{d\mathbb{P}}(a) - k_a^{\beta} \frac{d\mathbb{Q}}{d\mathbb{P}}(b) \right). \tag{60}$$

A priori, the sign of the expression above is ambiguous. If

$$\frac{d\mathbb{Q}}{d\mathbb{P}}(a) << \frac{d\mathbb{Q}}{d\mathbb{P}}(b) \tag{61}$$

then the risk-aversion channel dominates and countercyclical stocks trade at a premium. This premium for countercyclicality is ubiquitous in asset pricing theory. In fact, it is the only channel present in the CAPM—an asset's sensitivity to aggregate output or "beta" alone determines its price. In contrast, if

$$k_a^{\alpha} >> k_a^{\beta}, \tag{62}$$

then the "procyclical promises" channel dominates and procyclical stocks trade at a premium. This is the case when α-entrepreneurs can scale up and buy much more capital than β-entrepreneurs because debt capacity is very sensitive to cyclicality. This is likely to be the case when enforceability θ low, as pointed out in Corollary 1.

This positive side of procyclicality for corporate financing directly countervails against its negative side of procyclicality for risk sharing—collateral premium in my model replaces the insurance premium in models based on risk aversion. This may cast light on why models based only on risk aversion cannot explain the cross-section of asset prices (Fama and French (2004)). Further, it may cast light on why corporate leverage is decreasing in discount-rate beta but increasing in cash-flow beta (Maia (2010)): firms with high discount-rate beta have a cost of debt and therefore find it costly to lever up; in contrast, firms with high cash-flow beta can commit not to renegotiate their debt and therefore can expand their debt capacity.

5.3 New Testable Predictions and Further Empirical Content

In this section, I discuss some further empirical predictions of my model that have not yet been tested, as far as I know. I also discuss other empirical content that I have not touched on above.

The new empirical predictions I focus on here are driven by cross-sectional variation in the parameter θ. θ is the proportion of capital that entrepreneurs can divert, thus a low value of θ can represent low levels of asset tangibility or the weak legal enforcement. Now, recall that item (iv) of Corollary 1 says that debt capacity is more sensitive to cyclicality for high values of θ than for low values of θ. Given the correspondence between cyclicality and cash-flow beta established in Subsection 5.1 above, this suggests the following cross-sectional predictions:

(i) The correlation between leverage and cash-flow beta is higher for firms with low-tangibility assets than for firms with high-tangibility assets.

(ii) The correlation between leverage and cash-flow beta is higher for firms in countries with weak legal enforcement than for firms in countries with strong legal enforcement.

Further, an aggregate shock to θ should affect procyclical firms more than countercyclical firms.

(iii) The leverage and asset values of procyclical firms should be more sensitive to shocks to legal enforcement than those of countercyclical firms.

These comparative statics in θ also suggest some cross-country predictions about aggregate variables. Given that all fluctuations in the model are caused by enforcement frictions, countries with weaker enforceability should have more volatile aggregate output and larger business cycles. Under the assumption that the enforceability is lower in developing countries relative to developed countries, these predictions find support in the data. For example they are consistent with the following empirical findings:

(i) Output variability is negatively related to financial development (King and Levine (1993), Rajan and Zingales (1998)).

(ii) The amplitude of business cycles is larger in developing countries than in developed countries (Male (2011)).

6 Conclusion

This paper explores the role that procyclicality can play in mitigating enforcement frictions. I present a dynamic equilibrium model in which debt is subject to renegotiation. I find that procyclical firms can borrow more than countercyclical firms. The reason is that procyclical firms are less likely to renegotiate their debt repayments downward. This is because procyclical firms are more likely to renegotiate their debts in good times, when asset values are high. Thus, they put their creditors in a strong bargaining position during renegotiation, because asset liquidation serves as the threat that determines equilibrium repayments. The fact that procyclicality increases debt capacity affects asset prices. Procyclical capital is more expensive than countercyclical capital in equilibrium, because it allows firms to lever up. This price premium is a collateral premium for procyclical assets; it stands in contrast to the insurance premium for countercyclical assets that appears in models based on risk aversion.

The model generates time-series fluctuations in aggregate and firm-level variables. For example, capital prices, TFP, capital allocation, target leverage, constrained firms' leverage, and corporate borrowing are all endogenously procyclical in the model, consistent with stylized facts. The model generates these fluctuations even in the absence of technological shocks or capital accumulation. All time-series fluctuations result entirely from fluctuations in the efficiency of capital allocation, which, in turn, result from endogenous fluctuations in enforcement frictions.

The model suggests that taxing countercyclical entrepreneurs to subsidize procyclical entrepreneurs may improve utilitarian welfare, even though it amplifies the business cycle. This is because this tax-subsidy scheme transfers resources to firms that are less inhibited by enforcement frictions, leading to improved capital allocation in equilibrium.

A Proofs

A.1 Proof of Proposition 1

The proof follows from the fact that, in the first-best, each production good is put to its most productive use. Given that all players are risk-neutral, the most productive technology is simply the technology with the highest expected output. Since entrepreneurs' are always more productive than investors, α-entrepreneurs hold all α-capital and β-entrepreneurs hold all β-capital or $k_s^\alpha = K$ and $k_s^\beta = K$ for $s = a$ and $s = b$. The three statements of the proposition all follow, as outlined below.

Statement (i). Given that $k_s^\alpha = K$ and $k_s^\beta = K$, the definition of TFP (Definition 3) gives

$$\mathrm{TFP}_{s_t} \equiv \frac{\mathbb{E}\left[\alpha\big(k_{s_t}^\alpha\big) + \gamma\big(K - k_{s_t}^\alpha\big) + \beta\big(k_{s_t}^\beta\big) + \gamma\big(K - k_{s_t}^\beta\big)\right]}{2K} \tag{63}$$

$$= \frac{\mathbb{E}\left[\alpha\big(K\big) + \beta\big(K\big)\right]}{2K} \tag{64}$$

$$= A \tag{65}$$

since $\mathbb{E}\left[\alpha(k^\alpha)\right] = Ak^\alpha$ and $\mathbb{E}\left[\beta(k)\right] = Ak^\beta$ for all k^α and k^β.

Statement (ii). Expected output is

$$\text{expected output} = \mathbb{E}\left[\alpha\big(k_{s_t}^\alpha\big) + \gamma\big(K - k_{s_t}^\alpha\big) + \beta\big(k_{s_t}^\beta\big) + \gamma\big(K - k_{s_t}^\beta\big)\right] = 2AK. \tag{66}$$

Statement (iii). The price of α- and β-capital is determined by the pricing equation (MC) with entrepreneurs holding all the capital or $k_s^\tau = K$. Since $\gamma'(0) = A$,

$$R = \frac{\gamma'\big(K - k_s^\tau\big) + \bar{p}^\tau}{p_s^\tau} \tag{67}$$

$$= \frac{A + \bar{p}^\tau}{p_s^\tau} \tag{68}$$

for both capital goods and both states. Substituting in $\bar{p}^\tau = (p_a^\tau + p_b^\tau)/2$ and rearranging implies that p_a^τ and p_b^τ solve the following system:

$$\begin{cases} (2R-1)p_a^\tau = 2A + p_b^\tau, \\ (2R-1)p_b^\tau = 2A + p_a^\tau. \end{cases} \tag{69}$$

Solving this system gives $p_s^\tau = A/(R-1)$ for $\tau \in \{\alpha, \beta\}$ and $s \in \{a, b\}$. □

A.2 Proof of Lemma 1

The proof works by considering two cases, $p_a^\tau \geq p_b^\tau$ and $p_a^\tau < p_b^\tau$.

Case 1: $p_a^\tau \geq p_b^\tau$. The pricing equation (MC) says

$$p_a^\tau = \frac{\gamma'\big(K - k_a^\tau\big) + \bar{p}^\tau}{R}. \tag{70}$$

Since γ has decreasing returns to scale $\gamma'\big(K - k_a^\tau\big) < \gamma'(0) = A$. Further, we have that $\bar{p}^\tau < p_a^\tau$, since by the assumption of Case 1, $p_a > p_b$. We have

$$p_a^\tau \leq \frac{A + p_a^\tau}{R}. \tag{71}$$

Rearranging gives $p_a^\tau \leq A/(R-1)$.

Case 2: $p_a^\tau < p_b^\tau$. The proof in this case is identical to the above except the subscripts are reversed. □

A.3 Proof of Lemma 2

It suffices to show the following two statements:

(i) Given τ fails ($\tau(s)(k) = 0$), if the diversion constraint is satisfied (inequality (DC) holds), then the renegotiation constraint is satisfied (inequality (RC) holds).

(ii) Given τ succeeds ($\tau(s)(k) = 2Ak$), if the renegotiation constraint is satisfied (inequality (RC) holds), then the diversion constraint is satisfied (inequality (DC) holds).

Statement (i). This says that if $T(s) \leq \theta p_s^\tau k$, then $T(s) \leq p_s^\tau k$. This is always satisfied since $\theta \leq 1$.

Statement (ii). This says that if $T(s) \leq p_s^\tau k$, then $T(s) \leq 2Ak + \theta p_s^\tau k$. It suffices to show that

$$p_s^\tau k \leq 2Ak + \theta p_s^\tau k \tag{72}$$

or

$$p_s^\tau \leq \frac{2A}{1-\theta}. \tag{73}$$

Since, by Lemma 1, $p_s^\tau \leq A/(R-1)$, a sufficient condition for this is

$$\frac{A}{R-1} \leq \frac{2A}{1-\theta} \tag{74}$$

or, rearranging,

$$\theta \geq 3 - 2R. \tag{75}$$

This is always satisfied by Parameter Restriction 1. □

A.4 Proof of Proposition 2

In this proof I make use of the notations p_{succ} and p_{fail} for the price in the event of success and the price in the event of failure, as I do in Subsection 3.2. The proof has two steps. First, I show an upper bound on an entrepreneur's loan size ℓ in terms of his capital holdings k and the price of capital when he borrows p_t^τ. Second, I substitute in for k and p_t^τ from his budget constraint to derive the expression in the proposition.

Since the proof does not depend on the capital good or the type of the entrepreneur, I omit the superscript τ throughout.

Step 1. From equation (MC), we have that

$$\ell = \frac{\mathbb{E}[T]}{R} = \frac{T(a) + T(b)}{2R}. \tag{76}$$

Now, making use of the bounds on the repayment T in Lemma 2, we have

$$\ell \leq \frac{T(a)+T(b)}{2R} \tag{77}$$
$$= \frac{p_{\text{succ}}\, k + \theta p_{\text{fail}}\, k}{2R} \tag{78}$$
$$= \frac{\Big(\theta(p_{\text{succ}} + p_{\text{fail}}) + (1-\theta)p_{\text{succ}}\Big)\, k}{2R} \tag{79}$$
$$= \frac{\Big(2\theta\bar{p} + (1-\theta)p_{\text{succ}}\Big)\, k}{2R}. \tag{80}$$

Making use of Definition 2 and Definition 4, this says that

$$\ell \leq \frac{\big(\theta\rho + (1-\theta)\chi\big)p_t k}{R}. \tag{81}$$

Step 2. From the entrepreneur's budget constraint (equation (BC), we have that

$$p_t k = w + \ell. \tag{82}$$

Substituting for $p_t k$ in the upper bound for ℓ in equation (81) above, we have

$$\ell \leq \frac{\big(\theta\rho + (1-\theta)\chi\big)}{R}\big(w+\ell\big) \tag{83}$$

As long as $R > \theta\rho + (1-\theta)\chi$, we can rearrange and recover

$$\ell \leq \frac{\theta\rho + (1-\theta)\chi}{R - \theta\rho - (1-\theta)\chi} w \equiv \ell^{\max}, \tag{84}$$

as stated in the proposition.

It remains to show only that indeed $R > \theta\rho + (1-\theta)\chi$. This follows from the pricing equation (MC), which says

$$R = \frac{\gamma'(K-k_t) + \bar{p}}{p_t} \geq \frac{\bar{p}}{p_t} \equiv \rho. \tag{85}$$

Further, $\rho \geq \chi$, as you can see from equation (11). Since $R \geq \rho \geq \chi$, $R \geq \theta\rho + (1-\theta)\chi$, as desired. $\square$

A.5 Proof of Proposition 3

The result follows immediately from differentiation of the expression for $\ell^{\max}$ in Proposition 2.

A.6 Proof of Corollary 1

All of the statements in the corollary follow immediately from differentiation of the expression for $\ell^{\max}$ in Proposition 2, keeping in mind that $\rho > \chi$, which follows immediately from the definitions in Subsection 2.4.

A.7 Proof of Lemma 3

I prove this result in two steps. First, I prove a lemma that says that the first-best outcome is not attained. This ensures that $p_s^\tau < A/(R-1)$, i.e. the inequality in Lemma 1 is strict. Second, I show that entrepreneurs wish to scale up their projects, but their borrowing constraint binds, i.e. that they borrow to capacity.

Before launching into the formal proof, let me offer a word of intuition for the argument. If the first-best is attained, then entrepreneurs are marginal—i.e. they are *indifferent* to levering up more. Hence, I need to prove first that the first best is not attained. In other words, entrepreneurs are unable to borrow enough to achieve the first-best outcome, because they hit their borrowing constraints. After I have shown that the first best is not attained, I show that entrepreneurs borrow to capacity "trying to reach" the first-best level of investment.

LEMMA 6. (FIRST BEST NOT ATTAINED.) *The first-best outcome is not attained. For both types of entrepreneurs τ, $k_t^\tau < K$ at each date t.*

Proof. Suppose (in anticipation of a contradiction) that an entrepreneur holds all the capital at date t, $k_t^\tau = K$ for $\tau \in \{\alpha, \beta\}$. Thus, a τ-entrepreneur's budget constraint implies

$$K = \frac{w + \ell}{p_{s_t}^\tau}. \tag{86}$$

Observe that the right-hand side is bounded, since $\ell \leq \ell^{\max}$. Thus,

$$
\begin{aligned}
K &\leq \frac{w + \ell^{\max}}{p^{\tau}_{s_t}} && (87)\\
&= \frac{Rw}{Rp^{\tau}_{s_t} - \theta\bar{p}^{\tau} - (1-\theta)p^{\tau}_{\text{succ}}/2} && (88)\\
&\leq \frac{Rw}{Rp^{\tau}_{s_t} - \bar{p}^{\tau}}. && (89)
\end{aligned}
$$

The pricing equation (MC) gives an expression for the denominator above,

$$
Rp^{\tau}_{s_t} - \bar{p}^{\tau} = \gamma'(K - k^{\tau}_t) = \gamma'(0) = A, \tag{90}
$$

since $k^{\tau}_t = K$ by hypothesis. Substituting this into equation (89) implies

$$
K \leq \frac{Rw}{A}. \tag{91}
$$

This is contradicts Parameter Restriction 2. □

Since the first best is not attained, the pricing equation (MC) implies that $p^{\tau}_s < A/(R-1)$. I now show that this implies entrepreneurs borrow to capacity.

A τ-entrepreneur borrows ℓ at rate R to invest k in order to maximize

$$
\mathbb{E}\left[\tau(k) + p^{\tau}_{t+1}k - T\right] = Ak + \bar{p}^{\tau}k - R\ell \tag{92}
$$

subject to his budget constraint

$$
p^{\tau}_t k = w + \ell \tag{93}
$$

and the enforcement constraints as stated in Lemma 2. Substituting from the budget constraint gives the objective function

$$
\begin{aligned}
Ak + \bar{p}^{\tau}k - R\ell &= (A + \bar{p}^{\tau})\frac{w + \ell}{p_t} - R\ell && (94)\\
&= \left(\frac{A + \bar{p}^{\tau}}{p^{\tau}_t} - R\right)\ell + \frac{A + \bar{p}^{\tau}}{p^{\tau}_t}w. && (95)
\end{aligned}
$$

This is strictly increasing in ℓ as long as $(A + \bar{p}^{\tau})/p^{\tau}_t > R$. This is the case by Lemma 6 above, which implies that $p^{\tau}_t < A/(R-1)$. Thus, the entrepreneur sets the largest possible ℓ—he borrows to capacity.

A.8 Proof of Corollary 2

This follows immediately from Proposition 2 and Lemma 3: just substitute $\ell = \ell^{\max}$ into an entrepreneur's budget constraint in each state.

A.9 Proof of Lemma 4

This result is just an expression of the pricing equation (MC).

A.10 Proof of Proposition 4

Statement (i). For the proof of statement (i), see the proof of Proposition 5 below. There, I establish that α-entrepreneurs invest more than β-entrepreneurs in each state, i.e. $k_s^\alpha > k_s^\beta$. As a result, output in state a is greater than output in state b, since the α-technology pays off in state a and the β-technology pays off in state b.

The statements (ii)–(iii) in the proposition follow from the fact that $k_a^\tau > k_b^\tau$ for both types τ of entrepreneurs. I state this result as a lemma.

LEMMA 7. *All entrepreneurs invest more in state a than in state b, $k_a^\tau > k_b^\tau$ for $\tau \in \{a, b\}$.*

Proof. First note that if $k_s^\tau \geq k_{s'}^\tau$ then $p_s^\tau \geq p_{s'}^\tau$. This follows from the pricing equation and the fact that $\gamma'' < 0$. Specifically, equation (MC) implies that

$$p_s^\tau - p_{s'}^\tau = \gamma'(K - k_s^\tau) - \gamma'(K - k_{s'}^\tau) \geq 0, \tag{96}$$

whenever $k_s^\tau \geq k_{s'}^\tau$.

I prove the lemma first for α-capital and then for β-capital. The proofs have identical structures, both are by contradiction.

α-capital. Suppose (in anticipation of a contradiction) that $k_b^\alpha \geq k_a^\alpha$. Substituting in from equations (32) and (33) gives

$$\frac{2Rw_b}{(2R - \theta)p_b^\alpha - p_a^\alpha} \geq \frac{2Rw_a}{(2R - 1)p_a^\alpha - \theta p_b^\alpha} \tag{97}$$

or

$$\frac{w_b}{w_a} \geq \frac{2Rp_b^\alpha - p_a^\alpha - \theta p_b^\alpha}{2Rp_a^\alpha - p_a^\alpha - \theta p_b^\alpha} \geq 1 \tag{98}$$

since the hypothesis that $k_b^\alpha \geq k_a^\alpha$ implies $p_b^\alpha \geq p_a^\alpha$ as established above.

$$\frac{w_b}{w_a} \geq 1, \tag{99}$$

contradicting the assumption that $w_a > w_b$. Hence we conclude that $k_a^\alpha > k_b^\alpha$.

β**-capital.** Suppose (in anticipation of a contradiction) that $k_b^\beta \geq k_a^\beta$. Substituting in the from equations (34) and (35) gives

$$\frac{2Rw_b}{(2R-1)p_b^\beta - \theta p_a^\beta} \geq \frac{2Rw_a}{(2R-\theta)p_a^\beta - p_b^\beta} \tag{100}$$

or

$$\frac{w_b}{w_a} \geq \frac{2Rp_b^\beta - p_b^\beta - \theta p_a^\beta}{2Rp_a^\beta - p_b^\beta - \theta p_a^\beta} \geq 1 \tag{101}$$

since the hypothesis that $k_b^\beta \geq k_a^\beta$ implies $p_b^\beta \geq p_a^\beta$ as established above. Thus we have that

$$\frac{w_b}{w_a} \geq 1, \tag{102}$$

contradicting the assumption that $w_a > w_b$. Hence we conclude that $k_a^\beta > k_b^\beta$. □

Statement (ii). The fact that productivity is higher in state a than in state b follows from Lemma 7 above, which says that both types of entrepreneurs hold more capital in state a than in state b. Since entrepreneurs are more productive than investors, when entrepreneurs hold more capital, aggregate productivity is higher.

Statement (iii). The fact that prices are higher in state a than in state b follows immediately from Lemma 7 above (see the first step in the proof, circa equation (96).)

A.11 Proof of Lemma 5

To prove the result, just add the pricing equations for p_a^τ and p_b^τ in Lemma 4: since

$$Rp_a^\tau = \bar{p}^\tau + \gamma' (K - k_a^\tau) \tag{103}$$

and

$$Rp_b^\tau = \bar{p}^\tau + \gamma' (K - k_b^\tau), \tag{104}$$

it must be that

$$Rp_a^\tau + Rp_b^\tau = \Big(\bar{p}^{\,\tau} + \gamma'\,(K - k_a^\tau)\Big) + \Big(\bar{p}^{\,\tau} + \gamma'\,(K - k_b^\tau)\Big). \tag{105}$$

The left-hand side equals $2R\,\bar{p}^\tau$. Thus, rearranging gives the perpetuity expression in the lemma.

A.12 Proof of Proposition 5

The proof is by contradiction. I proceed in three steps. In Step 1, I show that the following three statements are equivalent: (i) $p_a^\alpha > p_a^\beta$, (ii) $p_b^\alpha > p_b^\beta$, and (iii) and $k_a^\alpha > k_b^\alpha$. In Step 2, I define the variable $\Delta^\tau := p_a^\tau - p_b^\tau$ and show that if $p_a^\beta \geq p_a^\alpha$ then $\Delta^\beta \geq \Delta^\alpha$. In Step 3, I suppose that $p_a^\beta \geq p_a^\alpha$ and show that it leads to a contradiction.

Recall that I make use of the "normalization" $w_b = 0$ for this proof, as I mention in the model setup. This normalization greatly simplifies the analysis because it implies that $k_b^\alpha = k_b^\beta = 0$. This follows directly from the budget constraint $p_s^\tau k_s^\tau = w_s + \ell_s^\tau$ and the expression for debt capacity in Proposition 2.

Step 1. Since $k_b^\alpha = k_b^\beta = 0$, Lemma 5 implies that

$$\bar{p}^\tau = \frac{\gamma'(K - k_a^\tau) + \gamma'(K)}{2(R-1)}. \tag{106}$$

Substituting this expression into the pricing equation (MC) gives

$$p_a^\tau = \frac{(2R-1)\gamma'(K - k_a^\tau) + \gamma'(K)}{2R(R-1)} \tag{107}$$

and

$$p_b^\tau = \frac{\gamma'(K - k_a^\tau) + (2R-1)\gamma'(K)}{2R(R-1)}. \tag{108}$$

The expressions for p_a^τ and p_b^τ in equations (107) and (108) both depend only on k_a^τ. The fact that $\gamma'' < 0$ implies that both p_a^τ and p_b^τ are increasing in k_a^τ. Thus, $p_a^\alpha > p_a^\beta$ if and only if $p_b^\alpha > p_b^\beta$ if and only if $k_a^\alpha > k_b^\alpha$.

Step 2. Define

$$\Delta^\tau := p_a^\tau - p_b^\tau. \tag{109}$$

Using equation (MC), this we recover an expression for Δ^τ in terms of k_a^τ,

$$\Delta^\tau = \frac{\gamma'(K - k_a^\tau) - \gamma'(K)}{R}. \tag{110}$$

The perpetuity expression from Lemma 5 implies

$$\bar{p}^\tau = \frac{\gamma'(K - k_a^\tau) + \gamma'(K)}{2(R-1)} \tag{111}$$

$$= \frac{2\gamma'(K) + R\Delta^\tau}{2(R-1)}. \tag{112}$$

This expression implies that $\bar{p}^\beta \geq \bar{p}^\alpha$ if and only if $\Delta^\beta \geq \Delta^\alpha$. The analysis in Step 1 implies that this is also equivalent to $p_a^\beta \geq p_a^\alpha$.

Step 3. Suppose (in anticipation of a contradiction) that $p_s^\beta \geq p_s^\alpha$. Thus, by Step 1, $k_a^\beta \geq k_a^\alpha$. Substituting in from the expressions for k_a^β and k_a^α in Corollary 2 this says that

$$\frac{2Rw_a}{(2R-\theta)p_a^\beta - p_b^\beta} \geq \frac{2Rw_a}{(2R-1)p_a^\alpha - \theta p_b^\alpha} \tag{113}$$

or

$$(2R-1)p_a^\alpha - \theta p_b^\alpha \geq (2R-\theta)p_a^\beta - p_b^\beta \tag{114}$$

Now, eliminate p_a^β and p_a^α from this inequality using $p_a^\beta = p_b^\beta + \Delta^\beta$ and $p_a^\alpha = p_b^\alpha + \Delta^\alpha$:

$$(2R-1)(p_b^\alpha + \Delta^\alpha) - \theta p_b^\alpha \geq (2R-\theta)(p_b^\beta + \Delta^\beta) - p_b^\beta \tag{115}$$

or

$$\big(2R - 1 - \theta\big)(p_b^\alpha - p_b^\beta) \geq (2R-1)(\Delta^\beta - \Delta^\alpha) + (1-\theta)\Delta^\beta. \tag{116}$$

The right-hand side is strictly positive since the hypothesis that $p_a^\beta \geq p_a^\alpha$ implies that $\Delta^\beta \geq \Delta^\alpha$ as established in Step 2 above and $\Delta^\beta > 0$ since $w_a > 0$. Thus, we have that

$$p_b^\alpha - p_b^\beta > 0, \tag{117}$$

contradicting the hypothesis that $p_b^\beta \geq p_b^\alpha$. We therefore conclude that $p_b^\beta > p_b^\alpha$. $\square$

A.13 Proof of Proposition 6

Differentiating the expression for welfare in equation (41), we see that increasing ϵ increases welfare if and only if $k_s^\alpha > k_s^\beta$, as established in Step 1 in the proof of Proposition 5 (Appendix A.12). $\square$

A.14 Proof of Proposition 7

The expression follows from substituting the expression for ℓ in equation (43) into the budget constraint (BC), and replacing $\bar{p}^\tau/p_t$ with ρ from Definition 2. $\square$

A.15 Proof of Proposition 8

This result follows from showing that the equations for the α- and β-capital are the same.

As a first step, note that either entrepreneurs borrow to capacity or the first best is attained (see the proof of Lemma 3). If the first best is attained, there is no collateral premium (Proposition 1), so it remains only to prove the statement for the case in which the first best is not attained and $\ell = \ell^{\max}$.

From the expression for $\ell^{\max}$ given in Proposition 7, the equations for capital holdings are

$$k_s^\tau = \frac{w_s + \ell^{\max}}{p_s^\tau} \tag{118}$$

$$= \frac{A + Rw_s}{Rp_s^\tau - \theta\bar{p}^\tau}. \tag{119}$$

Since the capital holding does not depend on the state in which the project pays off—it depends only on p_s^τ and $\bar{p}^\tau$, but not cyclicality—the pricing equations for α- and β-capital coincide (equation (MC)). Thus, there is no collateral premium. $\square$

A.16 Proof of Proposition 9

The expression follows from substituting the expression for ℓ in equation (47) into the budget constraint (BC), and the replacing $\bar{p}^\tau/p_t$ with ρ from Definition 2. $\square$

A.17 Proof of Proposition 10

This result follows from showing that the equations for the α- and β-capital are the same.

As in the proof of Proposition 10 above, note that either entrepreneurs borrow to capacity or the first best is attained (see the proof of Lemma 3). If the first best is attained, there is no collateral premium (Proposition 1), so it remains only to prove the statement for the case in which the first best is not attained and $\ell = \ell^{\max}$.

From the expression for $\ell^{\max}$ given in Proposition 9, the equations for capital holdings are

$$k_s^\tau = \frac{w_s + \ell^{\max}}{p_s^\tau} \tag{120}$$

$$= \frac{Rw_s}{Rp_s^\tau - \bar{p}^\tau}. \tag{121}$$

Since the capital holding does not depend on the state in which the project pays off—it depends only on p_s^τ and $\bar{p}^\tau$, but not cyclicality—the pricing equations for α- and β-capital coincide (equation (MC)). Thus, there is no collateral premium. □

A.18 Proof of Proposition 11

Since the capital holding does not depend on the state in which the project pays off—from equation (48), it depends only on p_s^τ but not cyclicality—the pricing equations for α- and β-capital coincide (equation (MC)). Thus, there is no collateral premium. □

A.19 Table of Notations

	Indices
$\tau \in \{\alpha, \beta, \gamma\}$	player indices and production functions—α- and β-entrepreneurs and investors γ
$s \in \{a, b\}$	Markov state
t	time index
	Prices
p_s^τ	price of τ-capital in state s
$\bar{p}^\tau$	average price of τ-capital
	Demand and Supply
k_s^τ	τ-capital demanded by τ-entrepreneurs in state s
K	total supply of each capital good
	Lending Contracts
$T^\tau(s)$	τ-entrepreneurs' promised repayment in state s
ℓ^τ	amount of fruit τ-entrepreneurs borrow
	Parameters
A	entrepreneurs' productivity
w_s	entrepreneurs' fruit endowment in State s
θ	enforceability parameter
R	reciprocal of investors' discount factor (gross interest rate)
	Other Variables
$\ell^{\max}$	debt capacity (Proposition 2)
ρ	expected holding return on capital (Definition 2)
χ	cyclicality (Definition 4)
TFP_s	productivity in state s (Definition 3)
$\mathrm{beta}_{\mathrm{CF}}^\tau$	cash-flow beta of a τ-entrepreneur (Subsection 5.1)
S_s^τ	price of τ stock in state s (extension in Subsection 5.2)
$\bar{S}_s^\tau$	average price of τ stock s (extension in Subsection 5.2)

References

Adrian, T. and H. S. Shin (2014). Procyclical leverage and value-at-risk. *Review of Financial Studies 27*(2), 373–403.

Akerlof, G. A. and P. M. Romer (1993). Looting: The economic underworld of bankruptcy for profit. *Brookings Papers on Economic Activity 24*(2), 1–74.

Albuquerque, R. and H. A. Hopenhayn (2004). Optimal lending contracts and firm dynamics. *The Review of Economic Studies 71*(2), 285–315.

Alvarez, F. and U. J. Jermann (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica 68*(4), 775–797.

Basu, S. and J. Fernald (2001). Why is productivity procyclical? why do we care? In *New Developments in Productivity Analysis*, NBER Chapters, pp. 225–302. National Bureau of Economic Research, Inc.

Begenau, J. and J. Salomao (2014). Firm financing over the business cycle. Technical report, Harvard Business School.

Benmelech, E. and N. K. Bergman (2008). Liquidation values and the credibility of financial contract renegotiation: Evidence from u.s. airlines. *The Quarterly Journal of Economics 123*(4), 1635–1677.

Benmelech, E. and N. K. Bergman (2009, March). Collateral pricing. *Journal of Financial Economics 91*(3), 339–360.

Benmelech, E. and N. K. Bergman (2011). Bankruptcy and the collateral channel. *The Journal of Finance 66*(2), 337–378.

Bernanke, B. and M. Gertler (1989). Agency costs, net worth, and business fluctuations. *The American Economic Review 79*(1), 14–31.

Biais, B. and A. Landier (2015). Endogenous agency problems and the dynamics of rents. Technical report, Toulouse School of Economics.

Bolton, P. and D. S. Scharfstein (1990, March). A theory of predation based on agency problems in financial contracting. *American Economic Review 80*(1), 93–106.

Campbell, J. Y., C. Polk, and T. Vuolteenaho (2010, January). Growth or glamour? fundamentals and systematic risk in stock returns. *Review of Financial Studies 23*(1), 305–344.

Choi, J. (2013). What drives the value premium?: The role of asset risk and leverage. *Review of Financial Studies*.

Cooley, T., R. Marimon, and V. Quadrini (2004, August). Aggregate consequences of limited contract enforceability. *Journal of Political Economy 112*(4), 817–847.

Dang, T. and Y. Wu (2015). Corporate investment over the business cycle. *Review of Finance*.

Danielsson, J., H. S. Shin, and J.-P. Zigrand (2004). The impact of risk regulation on price dynamics. *Journal of Banking & Finance 28*(5), 1069 – 1087.

Danielsson, J., H. S. Shin, and J.-P. Zigrand (2012). Procyclical leverage and endogenous risk. Technical report, London School of Economics.

DeMarzo, P. M. and M. J. Fishman (2007a). Agency and optimal investment dynamics. *Review of Financial Studies 20*(1), 151–188.

DeMarzo, P. M. and M. J. Fishman (2007b). Optimal long-term financial contracting. *Review of Financial Studies 20*(6), 2079–2128.

DeMarzo, P. M. and Y. Sannikov (2006). Optimal security design and dynamic capital structure in a continuous-time agency model. *The Journal of Finance 61*(6), 2681–2724.

Eisfeldt, A. L. and A. A. Rampini (2006). Capital reallocation and liquidity. *Journal of Monetary Economics 53*(3), 369 – 399.

Fama, E. F. and K. R. French (2004, Summer). The capital asset pricing model: Theory and evidence. *Journal of Economic Perspectives 18*(3), 25–46.

Geanakoplos, J. (1997). Promises, promises. In W. Arthur, S. Durlauf, D. Lane, and S. E. Program (Eds.), *The Economy as an Evolving Complex System II*, Advanced book program: Addison-Wesley. Addison-Wesley.

Geanakoplos, J. and F. Kubler (2004). Leverage, incomplete markets and crises. 2004 Meeting Papers 557, Society for Economic Dynamics.

Gromb, D. (1999). Renegotiation in debt contracts. Technical report, MIT.

Halling, M., J. Yu, and J. Zechner (forthcoming). Leverage dynamics over the business cycle. *Journal of Financial Economics*.

Hart, O. and J. Moore (1989, May). Default and renegotiation: A dynamics model of debt. Working Paper 520, MIT.

Hart, O. and J. Moore (1994, November). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics 109*(4), 841–79.

Hart, O. and J. Moore (1998, February). Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics 113*(1), 1–41.

IMF Research (2000). *World Economic Outlook, May 2000: Asset Prices and the Business Cycle*. World Economic Outlook. INTERNATIONAL MONETARY FUND.

Kehoe, T. and D. Levine (1993). Debt-constrained asset markets. *Review of Economic Studies 60*(4), 865–888.

King, R. G. and R. Levine (1993). Finance and growth: Schumpeter might be right. *The Quarterly Journal of Economics 108*(3), 717–737.

Kiyotaki, N. and J. Moore (1997, April). Credit cycles. *Journal of Political Economy 105*(2), 211–48.

Korajczyk, R. A. and A. Levy (2003, April). Capital structure choice: macroeconomic conditions and financial constraints. *Journal of Financial Economics 68*(1), 75–109.

Korteweg, A. and I. Strebulaev (2015). An empirical target zone model of dynamic capital structure. Technical report, Stanford University.

Lorenzoni, G. (2008). Inefficient credit booms. *Review of Economic Studies 75*(3), 809–833.

Maia, M. V. (2010). *Cash-Flow Risks, Financial Leverage and the Cross Section of Equity Returns*. Ph. D. thesis, University of Pennsylvania.

Male, R. (2011, May). Developing Country Business Cycles: Characterizing the Cycle. *Emerging Markets Finance and Trade 47*(0), 20–39.

Mironov, M. (2013). Taxes, theft, and firm performance. *The Journal of Finance 68*(4), 1441–1472.

Rajan, R. G. and L. Zingales (1998). Financial dependence and growth. *The American Economic Review 88*(3), 559–586.

Rampini, A. A. (2004). Entrepreneurial activity, risk, and the business cycle. *Journal of Monetary Economics 51*(3), 555 – 573.

Rampini, A. A. and S. Viswanathan (2010). Collateral, risk management, and the distribution of debt capacity. *The Journal of Finance 65*(6), 2293–2322.

Rampini, A. A. and S. Viswanathan (2013). Collateral and capital structure. *Journal of Financial Economics 109*(2), 466 – 492.

Roberts, M. R. and A. Sufi (2009, August). Renegotiation of financial contracts: Evidence from private credit agreements. *Journal of Financial Economics 93*(2), 159–184.

Ross, S. A. (1985). Debt and taxes and uncertainty. *The Journal of Finance 40*(3), 637–657.

Schwert, M. and I. Strubalaev (2015). Capital structure and systematic risk. Working paper, Stanford GSB.

Shleifer, A. and R. W. Vishny (1992, September). Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance 47*(4), 1343–66.

Williamson, O. E. (1988). Corporate finance and corporate governance. *The Journal of Finance 43*(3), 567–591.