CONTRACTING ON CREDIT RATINGS TO COMPETE FOR FLOWS*

Jason Roderick Donaldson† Giorgia Piacentino ‡

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Abstract

Delegated asset managers frequently refer to credit ratings in the contracts they offer their investors. However, regulators have advised against this. Why? In this paper, we present a model that suggests a new reason that delegated asset managers contract on credit ratings: contracting on ratings is a way for asset managers to compete for flows of investor capital. However, competition among asset managers triggers a race to the bottom: asset managers use ratings in their contracts even though it is socially inefficient. This inefficiency arises because contracting on ratings prevents risk sharing.

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†Washington University in St Louis; jasonrdonaldson@gmail.com.

‡Washington University in St Louis; piacentino@wustl.edu.

1 Introduction

Credit ratings provide public information about the risk of securities traded in capital markets. These ratings not only provide uninformed investors with new information, but also provide expert asset managers with verifiable signals that they can contract on.[1] Indeed, asset managers frequently make reference to ratings in the contracts they offer investors; for example, bond portfolio managers' contracts often depend on the ratings of the bonds in their portfolios.[2] However, regulators have cautioned against the use of ratings in contracts. For example, in 2010 the Financial Stability Board said that "Investment managers and institutional investors must not mechanistically rely on CRA ratings...[and should limit] the proportion of a portfolio that is CRA ratings-reliant." This leads to the main questions that we ask in the paper: Why do asset managers refer to credit ratings in their contracts? And are regulators right to suggest that they should not?

In this paper, we present a model that suggests a new reason that delegated asset managers contract on credit ratings: contracting on ratings is a way for asset managers to compete for flows of investor capital.[3] In other words, our model suggests that asset managers may contract on credit ratings as a way to attract investors, not only as a way to mitigate incentive problems with investors, as has been emphasized in the literature (see He and Xiong (2013) and Parlour and Rajan (2016)). Unfortunately, asset managers' competition for flows triggers a race to the bottom: asset managers use ratings in their contracts even though it is socially inefficient. This inefficiency arises because contracting on ratings

[1] One other paper that focuses on this role of credit ratings is Parlour and Rajan (2016), which studies how contracting on credit ratings may be useful to complete incomplete contracts, as discussed below.

[2] According to the Bank for International Settlements (2003), "it is common, for example, for fixed income investment mandates to restrict the manager's investment choices to investment grade credits"; that is to say that they restrict their portfolios to securities rated BBB– or higher by Standard & Poor's or Baa3 or higher by Moody's. Ashcraft and Schuermann (2008) affirm the importance of contracting on ratings, saying that "As investment mandates typically involve credit ratings, they comprise another point where the CRAs play an important role."

[3] Papers that show the importance of delegated asset managers' competition for flows include Berk and Green (2004), Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997, 1999), and Dasgupta and Piacentino (2015).

prevents risk sharing. Thus, our analysis suggests that the regulators may be right, and that asset managers' dependence on credit ratings should be limited.

Model preview. We present a model of delegated investment in which asset managers compete to manage a single investor's capital. Asset managers have better information about asset returns than the investor does. This informational advantage creates a reason for the investor to delegate his investment to an asset manager, since it allows the asset manager to allocate assets relatively efficiently in his portfolio. However, there is a misalignment of incentives between the asset manager and the investor, because they have different degrees of risk aversion. We make no assumption as to whether the asset manager or the investor is more risk averse, but we assume that they have utility functions in the same HARA class.[4]

The timing of the model is as follows. First, asset managers offer contracts to the investor. Each contract specifies a fixed fee and a (possibly non-linear) division of final wealth between the asset manager and the investor. The contract may depend on the asset manager's action, i.e. the "portfolio allocation," and the realization of a public signal, i.e. the "rating." Second, this rating is realized and the investor delegates investment to an asset manager. Third, this asset manager gets private information about asset returns and makes an investment decision, i.e. he allocates the investor's capital to a portfolio of assets.[5] Finally, the investment pays off and the final wealth is divided according to the asset manager's contract.

Results preview. We begin our analysis by studying the constrained-efficient outcome, i.e. the delegation contract and investment decision that maximize social welfare subject to the constraint that the asset manager's portfolio choice is incentive compatible. We show that the first-best outcome can be implemented by an affine contract, i.e. a fixed fee and a constant proportion of final wealth. This result follows from an application of a general result due to Wilson (1984): as long as utility functions are in the same HARA class, an affine contract

[4]See Subsection 2.1 for the precise definition of a class of HARA ("hyperbolic absolute risk-aversion") utility functions. This includes a relatively wide class of preferences; e.g., if everyone has CARA utility then our results apply for all risk-aversion parameters.

[5]Actually, we set up a general model that includes delegated portfolio choice as a special case. Since we are motivated by the use of credit ratings in delegated portfolio management contracts, we restrict attention to this application in the Introduction for illustrative purposes.

both implements efficient risk sharing *and* aligns incentives.[6] This benchmark implies that the classic trade-off between incentives and risk sharing is switched off in our setting, so contracting on the rating is not necessary to mitigate the incentive conflict between an asset manager and the investor.[7] Indeed, we chose this setup to study the effect of contracting on ratings beyond that of aligning incentives, which has been studied elsewhere in the literature (see Parlour and Rajan (2016)).

We then solve for the equilibrium of our model, in which asset managers may offer contracts contingent on the rating. Our first main result is that asset managers do make their contracts contingent on the rating, even though this does not mitigate the incentive problem. This is because contracting on the credit rating allows asset managers to compete to "attract portfolio flows," i.e. to be employed to manage the investor's capital. To see why this is the case, suppose that the investor employs an asset manager who offers a contract that does *not* depend on the rating. Due to competition, the asset manager breaks even on average, making zero expected profit. Thus, he takes losses for "bad" ratings and makes offsetting profits for "good" ratings. Since he gets strictly positive profits for these good ratings, there is room for another asset manager to undercut him by offering a contract *dependent on the rating* that offers a lower fee for these good ratings. Thus, our model suggests that flow competition, rather than incentive problems, may be the source of ratings-contingent asset management contracts.

Our second main result is that that competition among asset managers has a dark side: it prevents risk sharing and thus lowers welfare. Since asset managers contract on the rating to compete, they must break even *for each rating* in equilibrium. In other words, they get the

[6]To be specific, the main theorem in Wilson (1984) is that "If the sharing rule is efficient and linear [or affine] then truthful revelation is a Nash equilibrium." The efficient sharing rule is affine if and only if everyone has preferences in the same HARA class. We thus apply this result to an optimal contracting setting. It is also worth noting that Ross (1974) finds related results for the principal-agent problem, which are sometimes grouped under the heading of "The Principle of Similarity." Related results are also in Amershi and Stoeckenius (1983), Pratt and Zeckhauser (1989), and Wilson (1968).

[7]Note that this contrasts with the common intuition about principal-agent problems that the agent (the asset manager here) must bear an inefficiently high fraction of the risk in order to give him a strong incentive to act in the interest of the principal (the investor here). To understand why this trade-off is absent if and only if utility functions are in the same HARA class see footnote 17 below and Pratt (2000).

same payoff (of zero) no matter what the rating is. Thus, all of the rating risk is borne by the investor—there is no risk sharing between him and his asset manager. This result is related to Hirshleifer's (1971) result that if public information is revealed before markets open, it inhibits risk sharing. We find that if public information is *contractable* before asset managers are employed, it inhibits risk sharing. Thus, unlike in Hirshleifer (1971), the presence of a public signal inhibits risk sharing even if it contains no new information. Since contracting on the rating is not necessary to align incentives in our model, prohibiting contracting on the rating is Pareto improving.

Policy. Even though our model is stylized, we think that it provides a relevant perspective for policy. In the context of the model, regulators should limit the extent to which asset managers can contract on ratings to improve welfare, in line with the suggestion of the FSB quoted above. Regulators have already identified some potential risks of the mechanistic reliance on ratings, such as increased systemic risk; our analysis reveals another one: the mechanistic reliance on ratings may inhibit risk sharing. Thus, limiting contractual dependence on ratings may have the added benefit of preventing the race to the bottom which inhibits financial markets from performing one of their main functions, allowing investors to share risk. A regulator may implement this policy directly, by prohibiting asset managers from contracting on ratings, or indirectly, by encouraging ratings agencies to publicize information in a "soft" way that is difficult to contract on, e.g., they could release verbal reports rather than announce letter-based ratings.

Our finding that a regulator can improve risk sharing without changing the precision of ratings highlights the difference between our results and those in Hirshleifer (1971). In his setting, a regulator must decrease the precision of ratings to improve risk sharing; in our setting, in contrast, a regulator can limit contracting on ratings to improve risk sharing. However, *if contracting on ratings is possible*, then increasing their precision may have negative effects in our model, in line with Hirshleifer's findings. In particular, we show that under the assumption that asset managers do not get new information from ratings, coarser ratings Pareto dominate more precise ones. This is because more precise ratings allow asset managers to compete more aggressively—i.e. asset managers break even for more ratings realizations—which makes risk sharing more difficult. Of course, there are reasons that regulators may wish to pursue policies to make ratings more precise. Indeed, in our setting, if asset managers do get new information from ratings, then increasing ratings' precision has the benefit of improved investment efficiency. However, our findings show that, if ratings are contractable, then this benefit comes with the cost of worse risk sharing. Thus, we suggest that if regulators work to increase ratings' precision, it is all the more important that they also work to minimize the negative effects of increased ratings' precision on risk sharing, ideally by limiting contractual dependence on ratings.

Realism and portfolio constraints. In our model, we consider a general contracting environment. We allow for non-linear contracts that depend on the final wealth, on the asset manager's portfolio decisions, and on the rating. We find that the equilibrium contract has a simple and realistic form. It is affine, as are many real-world asset management contracts, such as the "two-and-twenty" contract.[8] Further, our equilibrium contract depends on credit ratings, as do many real-world contracts as well. Another realistic feature of the contract is that it is designed to compete for flows; attracting flows is arguably asset managers' primary objective as reflected by their compensation. However, there is one feature of real-world contracts that the equilibrium contract in our baseline model does *not* feature: portfolio constraints based on ratings.[9] However, in Section 5, we show that if the space of contracts that asset managers can offer is not so rich that they can offer the fully optimal contract, then portfolio constraints *do* emerge. Further, our main results also hold in this contracting environment: (i) asset managers use ratings-based portfolio constraints to attract investor flows, even if these constraints do not mitigate an incentive problem, and (ii) restricting

[8] See Agarwal, Daniel, and Naik (2009) and Elton, Gruber, and Blake (2003) for empirical studies of the forms of delegated asset managers' compensation contracts.

[9] For example, asset manager Threadneedle Investments provides an example of ratings-based investment mandates in its European Corporate Bond Fund prospectus: "The portfolio will not be more than 25% invested in securities rated AAA.... A maximum of 10% of the portfolio can be invested in below investment grade securities."

contracting on ratings increases welfare by enhancing risk sharing. A number of real-world frictions may prevent asset managers from offering fully optimal contracts, making portfolio constraints useful instead. For example, if a component of the asset manager's payoff is non-contractable—maybe due to reputation gains or perks—then contracting on final wealth alone may be insufficient to align incentives.

Other applications. In our model, we focus on the asset management industry, in which competition to attract new capital—"flows"—is a first-order determinant of compensation. However, the model may also cast light on other industries in which agents compete for business in the presence of asymmetric information. Insurance is one such industry. Our analysis suggests that if insurers are competitive, then contracting on all observables can actually prevent insurers from sharing risk with their clients—competition may prevent insurers from providing insurance. To mitigate this friction, a regulator may prohibit insurers from making contracts contingent on verifiable signals. Thus, our model provides theoretical support for current regulation of health insurance which limits making future insurance premiums contingent on realized health outcomes.[10]

Layout. In the remainder of the Introduction, we discuss some related literature. In Section 2, we present the model. In Section 3, we present the first-best and constrained efficient benchmarks. In Section 4, we solve the model and analyze welfare. In Section 5, we present an alternative specification of the model in which the space of feasible contracts is limited. Specifically, we suppose that asset management contracts are incomplete as in Parlour and Rajan (2016). We show that our results also hold in this setup. Section 6 is the conclusion. The Appendix contains all proofs and a table of notations.

1.1 Related Literature

Like us, Parlour and Rajan (2016) take the "contracting view" of credit ratings. They explore how asset managers contract on ratings to solve incentive problems when the ability to write

[10] See http://www.hhs.gov/healthcare/about-the-law/pre-existing-conditions/index.html.

"complete" optimal contracts is limited. In contrast, we explore how asset managers contract on ratings to compete for flows even when incentive problems can be solved by writing complete optimal contracts. Our two approaches are complementary. Indeed, in Section 5, we consider an alternative version of our model in which contracts are incomplete, as in Parlour and Rajan's model. We show our main results also hold in this setting: competition among asset manager leads them to contract on ratings, which reduces social welfare.

Our model may cast light on why asset managers contract on public information generally, not just credit ratings. Admati and Pfleiderer (1997) study the role of performance benchmarks in delegated asset management. They find benchmarks "cannot be easily rationalized...[and] are generally inconsistent with optimal risk sharing and do not lead to the choice of an optimal portfolio" (p. 323). Our results suggest that modeling competition for investor flows may help to rationalize portfolio benchmarks, even if these benchmarks have negative welfare effects in equilibrium. He and Xiong (2013) also study contracting on public information in delegated asset management. They focus mainly on how penalties based on tracking error mitigate the moral hazard problem between an asset manager and a fund family. In addition to these papers, our work builds on a large theory literature in delegated asset management, including Bhattacharya and Pfleiderer (1985), Dybvig, Farnsworth, and Carpenter (2010), Palomino and Prat (2003), and Stoughton (1993).

A number of theory papers study strategic information provision by credit ratings agencies, such as Bolton, Freixas, and Shapiro (2012), Bar-Isaac and Shapiro (2013), Boot, Milbourn, and Schmeits (2006), Donaldson and Piacentino (2012), Fulghieri, Strobl, and Xia (2013), Goel and Thakor (2015), Goldstein and Huang (2016), Kashyap and Kovrijnykh (2016), Doherty, Kartasheva, and Phillips (2012), Kartasheva and Yilmaz (2013), Kovbasyuk (2013), Manso (2014), Mathis, McAndrews, and Rochet (2009), Opp, Opp, and Harris (2013), Sangiorgi and Spatt (2016), and Skreta and Veldkamp (2009). We focus on contracting on ratings and model ratings as exogenous public signals, abstracting from strategic information transmission.

Kurlat and Veldkamp (2015) find that better public information, e.g. more precise ratings, decreases welfare due to a Hirshleifer-type effect. In their model, the result arises due to the effect on equilibrium price. We find a similar result due to the form of asset management contracts.

Finally, we point out that our paper is connected to the literature on contracting on public signals in bilateral principal-agent problems, such as Chaigneau, Edmans, and Gottlieb (2014a, 2014b), Kessler, Lülfesmann, and Schmitz (2005), Nalebuff and Scharfstein (1987), Cremer and McLean (1988), and Riordan and Sappington (1988). These papers do not feature contracting on public signals as a way to compete for investor flows, as our model does.

2 Model

In this section, we set up the model. The model constitutes an extensive game of incomplete information in which asset managers first compete in contracts in the hope of being employed by a single investor and then the employed asset manager takes an action on behalf of the investor. Final wealth is divided according to the contract of the employed asset manager.

The motivating portfolio choice application from the Introduction is a special case of our general model. We describe this special case explicitly in parallel to the general setup. This makes the model concrete and fixes ideas. It also allows us to provide closed-form solutions for the asset manager's action.

2.1 Players

There is a single investor with a unit wealth and von Neumann–Morgenstern utility u_{I} and there are at least two competitive asset managers with von Neumann–Morgenstern utility u_{A} and outside option $\bar{u}$. All asset managers are identical. The investor and the asset managers differ in their risk aversion. We make no assumption as to whether the investor or the asset manager is more risk averse but we assume that both utility functions are in the same class of hyperbolic absolute risk-aversion (HARA). Specifically, their absolute risk tolerances are affine with the same slope,

$$-\frac{u_i'(w)}{u_i''(w)} = a_i + bw \tag{1}$$

for $a_i > -bw$ for all w and for $i \in \{\mathrm{I}, \mathrm{A}\}$.[11] Note that this assumption imposes no restriction on the magnitude of the difference between the investor's and asset manager's risk aversions. The HARA class is a relatively large class of utility functions. For example, it contains all utility functions in the CARA class (exponential utility), which is commonly used in the literature—if $b = 0$ condition (1) implies that the investor and the asset managers have CARA utility with coefficients of absolute risk aversion a_{I}^{-1} and a_{A}^{-1}. Further, if $b = -1$ condition (1) implies that utility functions are quadratic,

$$u_i(w) = -\frac{1}{2}\big(a_i - w\big)^2. \tag{2}$$

In the application to portfolio choice that we explore below, we illustrate our results with quadratic utility, because it allows us to solve for the optimal contract and investment decisions in closed form.

Asset managers have private information, captured by a private signal σ, which is relevant for the investment decision. There is also a public signal or "rating" ρ that the investor can observe as well. An asset manager's signal σ and the rating ρ refer to the realizations of random variables $\tilde{\sigma}$ and $\tilde{\rho}$. We assume that asset managers' private information is better than the rating, in the sense that the sigma-algebra generated by $\tilde{\sigma}$ is finer than the sigma-algebra generated by $\tilde{\rho}$, $\mathrm{sigma}\,(\tilde{\rho}) \subset \mathrm{sigma}\,(\tilde{\sigma})$. In other words, asset managers do not learn from the rating. We make this assumption to switch off ratings' role in information-provision and focus on their role in contracting. However, this assumption is not strictly necessary for

[11] This ensures that the coefficients of absolute risk tolerance in condition (1) are always strictly positive or, equivalently, that the utility functions are always strictly increasing and strictly concave.

most of our results or policy prescriptions.[12] We do not impose any other restrictions on the distributions of $\tilde{\sigma}$ and $\tilde{\rho}$.

2.2 Actions and Contracts

The investor wishes to delegate his investment to an asset manager because the asset manager is better informed about the optimal action to undertake. The asset manager will take an action x that affects the distribution of wealth that the investor and asset manager will divide ex post. Thus, for each action x that the asset manager takes, final wealth is a random variable which we denote by $\tilde{w}(x)$. We assume that $\tilde{w}$ is a concave function of x for every state of the world.[13] The asset manager's signal provides him with information about the distribution of this random variable, making delegation valuable. However, the investor anticipates a misalignment of investment incentives since his risk aversion differs from the asset manager's. Contracts attempt to align incentives to mitigate this downside of delegating investment. Each asset manager offers a contract Φ that specifies his compensation. This contract may depend on the final wealth w, the rating ρ, and his action x, but not the private signal σ because it is not verifiable. In other words, the asset manager gets $\Phi(w, x, \rho)$ and the investor gets $w - \Phi(w, x, \rho)$.

Portfolio choice application. Our model is motivated by delegated asset management, in which case x represents an asset manager's portfolio choice decision. To fix ideas, consider the problem of allocating the initial unit of wealth between two assets, a risk-free asset with

[12] We use this assumption only in the proof of Proposition 5, which says that coarser information structures Pareto dominate finer ones. If we assumed that asset managers learned from the rating, the forces behind this result would not be affected, but the result would be attenuated due to a countervailing force: finer ratings might provide asset managers with information that would lead them to make better investment decisions. However, this force would not affect our policy prescription that ratings-contingent asset management contracts should be limited. This is because asset managers could still use the information in ratings to guide their decisions without contracting on it.

[13] This technical assumption allows us to solve the general model using the first-order approach. Note that it is not strictly necessary for our main results. In particular, our results hold in the in the portfolio choice application in which this assumption does not hold ($\tilde{w}$ is an affine function of x).

return R_f and a risky asset with return $\tilde{R}$. In this case, the final wealth is given by

$$\tilde{w}(x) = R_f + x\big(\tilde{R} - R_f\big). \tag{3}$$

In this case, we view asset manager's private information σ as the true standard deviation of $\tilde{R}$ and the rating ρ as an imperfect public signal about σ.[14]

Below, we use this portfolio choice application with quadratic utility (as in equation (2)) to provide an illustration of our general results. We make the following assumption to streamline this illustration: the mean return of the risky asset is known and is independent of the asset manager's private information σ and the rating ρ. This implies that

$$\mathbb{E}\left[\tilde{R}\,\middle|\,\tilde{\sigma} = \sigma\right] = \mathbb{E}\left[\tilde{R}\,\middle|\,\tilde{\rho} = \rho\right] = \mathbb{E}\big[\tilde{R}\big] =: \bar{R}.$$

With quadratic utility functions, we must restrict parameters to ensure that marginal utility is positive—i.e. that everyone always prefers more wealth to less. The following technical condition ensures this is the case in equilibrium:

$$\big(\bar{R} - R_f\big)\big(R - \bar{R}\big) \leq \sigma^2 \tag{4}$$

for all pairs (σ, R).[15]

[14] In reality, credit ratings indicate the default probability of a bond. Typically, increasing the default probability will simultaneously increase the variance and decrease the expectation of returns. To simplify the calculations, we assume that ratings are not informative about expected returns. This simplification is innocuous, because the full model shows that all the qualitative results below hold generally, including in situations in which ratings are informative about both the mean and variance.

[15] To ensure that marginal utility is positive, it must be that the investor's welath is always less than a_{I} and the asset managers' wealth is always less than a_{A}, as can be seen from the quadratic functional form. Given the equilibrium contract Φ, this implies that $w - \Phi(w) < a_{\mathrm{I}}$ and $\Phi(w) < a_{\mathrm{A}}$ for all possible realizations of w. Condition (4) ensures that these conditions are satisfied given the equilibrium contract Φ. Specifically, we solve the model assuming the conditions are satisfied and then check that they are satisfied as long as this condition (4) holds.

2.3 Timing

Formally, the timing is as follows:

1. Each asset manager offers a contract Φ.
2. The rating ρ is released.
3. The investor observes ρ and the profile of contracts and employs an asset manager.
4. The employed asset manager observes his private signal σ and takes an action x.
5. The final wealth is realized and it is distributed according to the contract Φ of the employed asset manager: the asset manager gets $\Phi(w, x, \rho)$ and the investor gets $w - \Phi(w, x, \rho)$.

Remark on timing. It is important for our results that the investor can condition his decision about which asset manager to employ on the asset managers' contracts and on the rating. This allows asset managers to contract on the rating as a way to compete for flows. We model this by assuming that (i) the investor observes the rating after asset managers offer contracts and (ii) the investor observes the rating before he chooses which asset manager to employ. These assumptions may seem stark, but we think that they have realistic interpretations: (i) captures the idea that investors are free to *switch* asset managers after observing the rating and (ii) captures the idea that ratings may *change*, leading investors to reallocate their capital. In other words, the assumptions that investors employ asset managers only once and that ratings are released only once are not crucial to the mechanism.

2.4 A Note on Notations

At times the contracting notation can be cumbersome, so we frequently suppress the arguments of some functions. In particular, the contract $\Phi = \Phi(w, x, \rho)$ is always a function of wealth w, the asset manager's action x, and the rating ρ, but we frequently write just Φ or $\Phi(w)$. Φ' denotes the partial derivative of Φ with respect to w, $\Phi' := \partial\Phi/\partial w$. The asset manager chooses the action given his signal σ, but we usually write just x for $x(\sigma)$.

A table summarizing our notations is in Appendix B.

3 Benchmarks: First Best and Constrained Efficiency

In this section, we solve for the first-best and constrained-efficient outcomes of the model. The main result of this section is that these outcomes coincide, i.e. the asset manager's incentive constraints alone do not move the outcome away from first-best. This result is useful to solve the model below.

3.1 First Best

We define the first-best outcome as the contract Φ and action x that maximize a weighted sum of utilities[16] of the investor and a representative asset manager (since all asset managers are identical, the utility of this "representative asset manager" can also represent the utilities of all asset managers). We normalize the welfare weight on the investor to one and denote the welfare weight on the asset manager by λ so the social welfare function is $u_{\mathrm{I}} + \lambda u_{\mathrm{A}}$. Thus, we define the first-best outcome as the solution of the program to

$$\text{maximize} \quad \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}(x) - \Phi\big) + \lambda u_{\mathrm{A}}\big(\Phi\big)\Big] \tag{5}$$

over all contracts $\Phi = \Phi(w, x, \rho)$ and all actions $x = x(\sigma)$. Note that, since x depends on σ and σ is a sufficient statistic for ρ, information frictions do not constrain this program. The

[16] We define any outcome on the Pareto frontier as first best for a given welfare weight. In contrast, it is common in the contract-theory literature to define the first-best outcome as the one that maximizes the payoff of the principal (the investor) subject to the participation constraint of the agent (the asset manager); see, e.g., Bolton and Dewatripont (2005). This is a special case of our definition (choose the welfare weight λ so that the asset manager's expected utility equals his reservation utility $\bar{u}$). We use the more general definition because the results we get here are useful below (cf. Proposition 5).

next proposition characterizes the solution of the program.

PROPOSITION 1. (FIRST BEST.) *The first-best contract Φ_{fb} is affine and given by*

$$\Phi_{\text{fb}} = \begin{cases} \dfrac{a_{\text{I}} - \lambda^{-b} a_{\text{A}} + bw}{b\left(1 + \lambda^{-b}\right)} & \text{if } b \neq 0, \\[2ex] \dfrac{a_{\text{A}}}{a_{\text{A}} + a_{\text{I}}}\Big(a_{\text{I}} \log \lambda + w\Big) & \text{if } b = 0, \end{cases} \tag{6}$$

and the first-best action $x_{\text{fb}} = x_{\text{fb}}(\sigma)$ solves the first-order condition

$$\frac{\partial}{\partial x}\, \mathbb{E}\Big[u_{\text{I}}\big(\tilde{w}(x) - \Phi_{\text{fb}}\big) + \lambda u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\big|\, \tilde{\sigma} = \sigma\Big] = 0$$

for each realization of σ.

Proof. The proof is in Appendix A.1. □

The first-best contract is affine in wealth, like real-world asset management contracts that often constitute a fixed fee and a constant proportion of profits. Further, the contract depends only on the total final wealth w and the welfare weight λ. It does not depend on the rating. We cannot solve for the action x in closed form in the general model, but in the application to portfolio choice with quadratic utility, we can give a closed-form expression for the portfolio weight x. This is the next corollary.

COROLLARY 1. *In the portfolio-choice application with quadratic utility, the first best contract and investment are given by*

$$\Phi_{\text{fb}} = a_{\text{A}} + \frac{w - a_{\text{I}} - a_{\text{A}}}{1 + \lambda}$$

and

$$x_{\text{fb}} = \frac{\big(\bar{R} - R_f\big)\big(a_{\text{I}} + a_{\text{A}} - R_f\big)}{\sigma^2 + \big(\bar{R} - R_f\big)^2}. \tag{7}$$

Proof. The proof is in Appendix A.2. □

3.2 Constrained Efficiency

We now turn to the constrained-efficient outcome. This is the allocation that maximizes the expectation of the same social welfare function $u_{\mathrm{I}} + \lambda u_{\mathrm{A}}$ as the first-best outcome, but the asset manager's action x must be incentive-compatible given the contract Φ. Namely, the action x maximizes the asset manager's payoff rather than the social welfare function, given the contract Φ. Thus, we define the constrained-efficient outcome as the solution to the program to

$$\begin{cases} \text{maximize} \;\; \mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}(x) - \Phi\big) + \lambda u_{\mathrm{A}}\big(\Phi\big) \,\middle|\, \tilde{\rho} = \rho\right] \\ \\ \text{subject to} \;\; x \in \arg\max \left\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big) \mid \tilde{\sigma} = \sigma\Big]\right\} \end{cases} \tag{8}$$

over all contracts $\Phi = \Phi(w, x, \rho)$. The next proposition characterizes the solution to this program, which coincides with the first-best outcome.

Proposition 2. (Constrained-efficient outcome is first best.) *The constrained-efficient outcome coincides with the first-best outcome (as given in Proposition 1 and Corollary 1 above).*

Proof. The proof is in Appendix A.3. □

This proposition says that if the asset manager is compensated according to first-best contract, then the first-best action is incentive compatible. This result follows from the fact that the first-best contract is affine. The contract serves two roles: to share risk and align incentives. However, when the contract that implements the first-best risk sharing is affine, it also aligns incentives perfectly.[17]

[17] We briefly sketch why this is the case mathematically by comparing the equations for efficient risk sharing and incentive alignment. The condition for efficient risk sharing is that $u_{\mathrm{I}}(w - \Phi) + \lambda u_{\mathrm{A}}(\Phi)$ is maximized for each w, or that the ratio of marginal utilities is

$$\frac{u_{\mathrm{I}}'(w - \Phi(w))}{u_{\mathrm{A}}'(\Phi(w))} = \lambda. \tag{9}$$

Now recall that two utility functions induce the same choices—i.e. incentives are perfectly aligned—if one

4 Results

In this section, we solve the model and prove our main results. We first show that asset managers offer contracts that depend on the rating, even though contracting on the rating is not necessary to mitigate the incentive problem between the investor and an asset manager. Next we solve for the equilibrium contract. We do this by reformulating the model in a principal-agent framework and using the method of Lagrange multipliers. Finally, we show that increasing the precision of the rating decreases welfare.

4.1 Competition Is Rating-by-Rating

We now turn to our first main result, that asset managers actively contract on the rating to compete for investor flows. They use contracting on the rating to compete "rating-by-rating" and thus break even for every realization of $\tilde{\rho}$.

PROPOSITION 3. *The contract $\Phi = \Phi(w, x, \rho)$ of the employed asset manager depends on the rating ρ. The asset manager breaks even for each realization ρ of the rating, or*

$$\mathbb{E}\left[u_{\mathrm{A}}\big(\Phi\big) \,\big|\, \tilde{\rho} = \rho\right] = \bar{u}.$$

Proof. The proof is in Appendix A.4. □

Asset managers are competitive, so it should not be surprising that they receive their reservation utility in equilibrium. The takeaway from Proposition 3 above is that asset managers receive their reservation utility for *every rating ρ*. In other words, there cannot be an equi-

utility function is an affine transformation of the other. In our context, this is the case if there are constants C_1 and C_2 such that $u_{\mathrm{I}}(w - \Phi(w)) = C_1 u_{\mathrm{A}}(\Phi(w)) + C_2$. Differentiating this condition with respect to wealth says the ratio of marginal utilities must be

$$\frac{u_{\mathrm{I}}'(w - \Phi(w))}{u_{\mathrm{A}}'(\Phi(w))} = \frac{C_1 \Phi'(w)}{1 - \Phi'(w)}$$

Equating the right-hand sides of the equation above and of equation (9) says that there is efficient risk sharing and incentive alignment only if Φ' is constant, i.e. Φ is affine.

librium in which asset managers break even in expectation over all possible realizations of $\tilde{\rho}$ unless they break even for every realization of $\tilde{\rho}$. To see this, observe that if an asset manager did not break even for every rating, but only in expectation, then an asset manager who receives less than his reservation utility $\bar{u}$ for some rating must receive more than $\bar{u}$ for another rating. But since the asset manager is getting strictly more than $\bar{u}$ for this rating, there is room for a competing asset manager to profitably undercut him by offering a contract *dependent on the rating* that allocates more of the surplus to the investor.

The argument above glosses over one subtlety: when a competing asset manager offers a contract dependent on the rating to attract the investor, this contract may not only reallocate surplus toward the investor for certain ratings, but may also distort the manager's incentives and therefore change the action x. In the proof, we show that the competing asset manager can offer a "calibrated contract" that indeed undercuts the original asset manager's contracts while inducing him to choose the same action x. Specifically, if the original asset manager offers Φ, then for $\varepsilon > 0$ the calibrated contract $\Phi_\varepsilon(w) := u_{\mathrm{A}}^{-1}\big(u_{\mathrm{A}}\big(\Phi(w)\big) - \varepsilon\big)$ induces the same choice of x as Φ but allocates more of the surplus to the investor.

4.2 Equilibrium Contract

We now solve for the equilibrium contract by reformulating the model in a principal-agent framework. In this framework, the investor is the principal and the employed asset manager is the agent. The investor maximizes his utility over all contracts Φ subject to the asset manager's incentive constraint and participation constraint. The twist on the classical principal-agent setting is that the asset manager's participation constraint must bind for each rating ρ, since, by Proposition 3, asset managers contract on the rating to attract flows and thus must break even for each rating.

Lemma 1. *For each rating ρ, the contract of the employed asset manager solves the following* *principal-agent problem:*

$$\begin{cases} \text{Maximize } \ \mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big)\mid\tilde{\rho}=\rho\Big] \\ \\ \text{subject to } \ \mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big)\mid\tilde{\rho}=\rho\Big]=\bar{u} \text{ and} \\ \\ x\in\arg\max\Big\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big)\mid\tilde{\sigma}=\sigma\Big]\Big\} \end{cases} \tag{10}$$

over all contracts $\Phi=\Phi(w,x,\rho)$.

Next we solve the principal-agent problem in the program (10) for each rating ρ. We eliminate the asset manager's participation constraint using the method of Lagrange multipliers, but do not eliminate his incentive constraint. Since the asset manager breaks even for each rating, the participation constraint depends on ρ and thus so does the Lagrange multiplier on the constraint. We denote this Lagrange multiplier by λ_ρ and re-write the principal-agent problem as follows:

$$\begin{cases} \text{maximize } \ \mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi\big)+\lambda_\rho u_{\mathrm{A}}\big(\Phi\big)\,\Big|\,\tilde{\rho}=\rho\right] \\ \\ \text{subject to } \ x\in\arg\max\Big\{\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big)\Big|\,\tilde{\sigma}=\sigma\Big]\Big\}. \end{cases} \tag{11}$$

over all contracts $\Phi=\Phi(w,x,\rho)$, where the Lagrange multiplier λ_ρ makes the asset manager's participation constraint bind, i.e.

$$\mathbb{E}\Big[u_{\mathrm{A}}\big(\Phi\big)\mid\tilde{\rho}=\rho\Big]=\bar{u} \tag{12}$$

for all ρ. Now observe that the program (11) corresponds exactly to the program (5) above for the constrained-efficient outcome. The Lagrange multiplier λ_ρ on the asset manager's participation constraint corresponds to the welfare weight λ in the program for the constrained-efficient outcome. The twist is that the Lagrange multiplier depends on the rating ρ, because the asset manager must break even for each rating. Since we have already solved for the constrained-efficient outcome, we can apply our results above to express the equilibrium contract as a function of the Lagrange multiplier λ_ρ.

Proposition 4. (Equilibrium outcome in terms of Lagrange multiplier λ_ρ.) *The equilibrium contract is given by*

$$\Phi = \Phi_{\text{fb}}\Big|_{\lambda=\lambda_\rho} = \frac{1}{b(1+\lambda_\rho^{-b})}\Big(a_{\text{I}} - \lambda_\rho^{-b} a_{\text{A}} + bw\Big).$$

The asset manager chooses the first-best action $x = x_{\text{fb}}$. This corresponds to the first-best outcome from Proposition 1 with the social welfare weight λ replaced by the Lagrange multiplier λ_ρ. (Thus, since λ_ρ depends on ρ, the equilibrium contract depends on the rating, whereas the first-best contract does not.)

Proof. The proof is in Appendix A.6. □

4.3 Coarser Ratings Are Pareto-superior

Having established that asset managers offer contracts that depend on the rating, we now turn to the question of how this dependence affects welfare. We can focus on the investor's payoff alone because asset managers are competitive and so their payoff is always equal to their reservation utility $\bar{u}$. Now, given a rating ρ, we have

$$\text{investor's expected payoff given } \rho \; = \mathbb{E}\Big[u_{\text{I}}\big(\tilde{w}(x) - \Phi_\rho\big) \,\Big|\, \tilde{\rho} = \rho\Big].$$

Notice that we have modified our notation slightly and denoted the equilibrium contract given a rating ρ by Φ_ρ.[18] Using the law of iterated expectations, we can write the investor's

[18]Notice also that we have omitted the incentive constraint. This is without loss of generality since the asset manager takes the first-best action under the equilibrium contract (by Proposition 4).

ex ante payoff as

$$\mathbb{E}\left[\mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi_{\rho}\big)\,\Big|\,\tilde{\rho}=\rho\Big]\right]=\mathbb{E}\Big[u_{\mathrm{I}}\big(\tilde{w}-\Phi_{\tilde{\rho}}\big)\Big].$$

This expression reveals that, because the contract $\Phi_{\tilde{\rho}}$ depends on the random variable $\tilde{\rho}$, the investor's payoff is varying with the rating. In other words, the investor bears the risk over the rating. Because the investor is risk-averse, this decreases his welfare. Further, increasing the information contained in the rating—making the rating "finer"—only increases the risk that the investor bears over its outcome.[19] Indeed, coarser ratings Pareto dominate finer ratings, as we formalize in the next proposition.

Proposition 5. (Coarser ratings Pareto-dominate finer ratings.) *If ratings are generated by the random variables $\tilde{\rho}_c$ and $\tilde{\rho}_f$ such that $\tilde{\rho}_c$ is "coarser" than $\tilde{\rho}_f$—i.e.* $\mathrm{sigma}(\tilde{\rho}_c)\subset\mathrm{sigma}(\tilde{\rho}_f)$*—then the ex ante utility of the investor and all asset managers is at least as high given $\tilde{\rho}_c$ as given $\tilde{\rho}_f$. (Typically the investor is strictly better off.)*

Proof. The proof is in Appendix A.7. In Appendix A.8 we provide a more direct alternative proof for the application to portfolio choice with quadratic utility. □

The mechanism behind this result hinges on Proposition 3. Because competition makes asset managers break even "rating-by-rating," there is one participation constraint for each rating. Thus, with a finer rating structure there are more possible ratings (realizations of $\tilde{\rho}$) and, thus, more constraints on the investor's objective. Because we know from Proposition 2 that the efficient action is always taken, these constraints only restrict risk sharing between the investor and the asset manager. Hence, a finer ratings structure shuts down risk sharing and reduces welfare.

One way to get the intuition for this result is to contrast two situations: (i) ρ is complete noise versus (ii) ρ fully reveals σ. In (i), the asset manager's participation constraint must bind in expectation over σ, whereas in (ii) it must bind for every realization of σ. Given that

[19] Ratings have been made finer in reality. For example, in 1982 Moody's added numerical modifiers to its ratings, thereby refining its ratings partition. See Kliger and Sarig (2000) for analysis of this event. (Thanks to Joel Shapiro for drawing our attention to this.)

the investor is risk-averse, optimal risk sharing entails that the asset manager's utility varies with σ (given that the investor's utility must vary with σ). Thus forcing the asset manager to have the same utility for all realizations of σ leads to a sub-optimal outcome—contracting on the rating is detrimental to risk sharing.

This result is closely related to the Hirshleifer (1971) effect, by which information destroys gains from risk sharing. Two differences between our finding and Hirshleifer's are (i) our result obtains only with competing asset managers, whereas Hirshleifer's would with a single asset manager and (ii) our result depends only on contracting on public information, even before it is released, whereas Hirshleifer (1971)'s relies on trading after public information is released. This distinction also points to the importance of the sequencing of events in our model. Because asset managers offer contracts *before* the rating is released, they have the potential to share risk with the investor. However, this is undermined by asset manager's contracting on the rating to compete for flows. Further, the more precise the rating is, the less risk sharing there is in equilibrium.

5 Incomplete Contracts and Portfolio Constraints

In this section, we analyze an alternative version of the model. This analysis serves two purposes. First, it shows that our results are not driven by our specific modeling assumptions, but rather follow from the fact that asset managers offer contracts to attract investor flows. Second, it shows that competition for flows can generate contracts that feature rating-based portfolio constraints—like many real world asset management contracts—even if they do not serve to mitigate any incentive problems between asset managers and the investor. The version of the model that we present below is roughly based on the incomplete contracting setup in Parlour and Rajan (2016).

5.1 Setup

As in the baseline model, asset managers first offer contracts, then the rating ρ is released, and the investor employs an asset manager. The employed asset manager observes the private signal σ and then takes the action x. Here, we allow asset managers to offer contracts only in the form of a flat fee Φ and a portfolio constraint which specifies a restriction on x as a function of ρ. Further, for simplicity, we assume that σ and x are both binary, $\sigma \in \{0, 1\}$ and $x \in \{0, 1\}$. Denote the probability that $\sigma = 1$ by $p := \mathbb{P}\{\sigma = 1\}$. Further, σ and x completely determine the final wealth w, where w has the following binary distribution:

$$\tilde{w}(x) = \begin{cases} w_H & \text{if } x = \sigma, \\ w_L & \text{if } x \neq \sigma, \end{cases}$$

where $w_H > w_L$. Thus, $x = \sigma$ is the "right" action for the asset manager to take for each signal σ.[20]

Here we impose a specific structure on how the rating ρ is informative about σ. We assume that ρ either reveals that $\sigma = 1$ or does not reveal σ to the investor. Specifically, ρ is generated by a binary random variable $\tilde{\rho} \in \{\rho_1, \rho_\varnothing\}$ such that

$$\mathbb{P}\{\tilde{\sigma} = 1 \,|\, \tilde{\rho} = \rho_1\} = 1 \quad \text{and} \quad \mathbb{P}\{\tilde{\sigma} = 1 \,|\, \tilde{\rho} = \rho_\varnothing\} =: p_{\rho_\varnothing} > 0. \tag{13}$$

So far, this setup is a special case of the baseline model, with a restricted set of contracts. We depart from the baseline model with the following two assumptions. (i) Asset managers are risk-neutral, but they always prefer to take the same action—for example, asset managers always prefer to invest in the risky asset, independently of the investor's preferences. We model this by assuming that the asset manager receives private benefit B whenever he chooses $x = 1$. (ii) Despite this potential misalignment of incentives, the asset manager always takes

[20] Since w is not random given σ, our assumption that the contractual transfer is just a fixed fee Φ does not impose a restriction on risk sharing.

the (feasible) action that is in the best interest of the investor, for example because he obeys his fiduciary responsibility to do so. We make this assumption to shut down the role that contracts can play in aligning incentives, so as to emphasize that our results come from the role that contracts can play in asset managers' competition for flows.

Finally, we impose the technical condition that the asset manager's private benefit is small relative to the gains from taking the right action. Specifically,

$$B < \frac{p_{\rho\varnothing}}{1 - p_{\rho\varnothing}} \big(w_H - w_L \big). \tag{14}$$

5.2 Analysis without Portfolio Constraints

We first consider the benchamark case in which asset managers cannot offer portfolio constraints. Asset managers always act in the interest of the investor, so play $x = \sigma$, and receive private benefit B from choosing $x = 1$. Thus, the employed asset manager gets

$$\text{asset manager's payoff } = \Phi_{\text{nc}} + pB$$

on average,[21] where the subscript "nc" stands for "no constraints." Since asset managers are competitive, they offer Φ so that their break-even constraint binds: $\bar{u} = \Phi_{\text{nc}} + pB$ or

$$\Phi_{\text{nc}} = \bar{u} - pB.$$

We now turn to the investor's payoff. Since, by assumption, the asset manager always takes the efficient action, final wealth is always equal to w_H. Further, the asset manager charges a fixed fee. Thus, the investor's payoff is riskless. His ex ante expected utility is thus

$$\mathbb{E}\left[u_{\text{I}}\big(\tilde{w}(x) - \Phi_{\text{nc}} \big) \right] = u_{\text{I}}\big(w_H - \bar{u} + pB \big). \tag{15}$$

[21] In writing the break-even condition, we have implicitly restricted attention to equilibria in which the investor always uses the same tie-breaking rule when he employs an asset manager: if he is indifferent among asset managers, he always employs the same asset manager. This is without loss of generality.

5.3 Analysis with Portfolio Constraints

We now study the case in which asset managers can offer portfolio constraints. We show that they will use them to attract investor flows, which decreases welfare. We show this by contradiction. Suppose there is an equilibrium in which asset managers offer contracts without portfolio constraints as above, so the fee is given by Φ_{nc} above. Now consider a "deviant" asset manager who deviates by offering a contract with a slightly lower fee, say $\Phi_{\text{c}} := \Phi_{\text{nc}} - \varepsilon$, but with a portfolio constraint committing to choose $x = 1$ whenever $\tilde{\rho} = \rho_1$ and to choose $x = 0$ otherwise.[22] Given $\tilde{\rho} = \rho_1$, the investor knows that $\tilde{\sigma} = 1$ (by the assumption (13)) and therefore this deviant asset manager will take the right action at the lower fee $\Phi_{\text{c}} < \Phi_{\text{nc}}$. Thus, the investor employs him when $\tilde{\rho} = \rho_1$, but not when $\rho = \rho_{\varnothing}$, since in that case he may not take the right action.[23] The deviant asset manager gets $\Phi_{\text{c}} + B$ when $\tilde{\rho} = \rho_1$ and $\bar{u}$ when $\tilde{\rho} = \rho_{\varnothing}$ and so his expected payoff is

$$\text{deviant asset manager's payoff } = \mathbb{P}\left\{\tilde{\rho} = \rho_1\right\}\Big(\Phi_{\text{c}} + B\Big) + \mathbb{P}\left\{\tilde{\rho} = \rho_{\varnothing}\right\}\bar{u}.$$

This is a profitable deviation whenever the payoff above is greater than $\bar{u}$ or

$$0 < \mathbb{P}\left\{\tilde{\rho} = \rho_1\right\}\big(\Phi_{\text{c}} + B - \bar{u}\big).$$

[22] To interpret this as a realistic portfolio constraint, we view $x = 1$ as investing in risky corporate bonds and $x = 0$ as investing in safe sovereign bonds and we view $\tilde{\rho} = \rho_1$ as corporate bonds receiving a high rating. Thus, this portfolio constraint corresponds to an investment mandate to invest only in highly-rated bonds.

[23] We assume that the difference in fees $\Phi_{\text{nc}} - \Phi_c = \varepsilon$ is sufficiently small so that fee savings do not induce the investor to employ the deviant asset manager unless he knows that he will choose the right action. Note that when $\rho = \rho_{\varnothing}$ the deviant asset manager is constrained to choose the wrong action when $\tilde{\sigma} = 1$.

Substituting in $\Phi_{\text{c}} = \Phi_{\text{nc}} - \varepsilon$ and $\Phi_{\text{nc}} = \bar{u} - pB$ from above, we can reduce this condition as follows:

$$\begin{aligned}
0 &< \mathbb{P}\{\tilde{\rho} = \rho_1\}\big(\Phi_{\text{nc}} - \varepsilon + B - \bar{u}\big) \\
&= \mathbb{P}\{\tilde{\rho} = \rho_1\}\big((\bar{u} - pB) - \varepsilon + B - \bar{u}\big) \\
&= \mathbb{P}\{\tilde{\rho} = \rho_1\}\big((1-p)B - \varepsilon\big).
\end{aligned}$$

This holds as long as $(1-p)B > \varepsilon$. Since ε can be arbitrarily small, there is always a profitable deviation to a contract with portfolio constraints. This is because it allows the deviant asset manager to be employed—i.e. attract flows—when $\rho = \rho_1$. This leads to the first main result of this section.

Proposition 6. (Asset managers contract on ratings.) *There is no equilibrium in which no asset manager contracts on the rating. Asset managers use rating-contingent portfolio constraints to attract investor flows.*

We now characterize the equilibrium in this case in which asset managers can offer portfolio constraints and analyze how portfolio constraints affect investor welfare. In equilibrium, asset managers offer two kinds of contracts. At least one asset manager offers a rating-contingent contract like the deviant asset manager above. Since he is employed only when $\tilde{\rho} = \rho_1$, he always gets the private benefit B from choosing $x = 1$. Denoting his fee by Φ_{ρ_1}, his break-even condition is

$$\bar{u} = \mathbb{P}\{\tilde{\rho} = \rho_1\}\big(\Phi_{\rho_1} + B\big) + \mathbb{P}\{\tilde{\rho} = \rho_\varnothing\}\,\bar{u}.$$

Thus,

$$\Phi_{\rho_1} = \bar{u} - B.$$

Other asset managers offer non-contingent contracts like the contract without portfolio constraints above. However, an asset manager with a non-contingent contract is employed only if $\rho = \rho_\varnothing$. Denoting his fee by $\Phi_{\rho_\varnothing}$, his break-even condition is

$$\bar{u} = \mathbb{P}\left\{\tilde{\rho} = \rho_1\right\} \bar{u} + \mathbb{P}\left\{\tilde{\rho} = \rho_\varnothing\right\} \Big(\Phi_{\rho_\varnothing} + \mathbb{P}\left\{\tilde{\sigma} = 1 \,|\, \tilde{\rho} = \rho_\varnothing\right\} B\Big).$$

Thus, using the notation $p_{\rho_\varnothing} \equiv \mathbb{P}\left\{\tilde{\sigma} = 1 \,|\, \tilde{\rho} = \rho_\varnothing\right\}$,

$$\Phi_{\rho_\varnothing} = \bar{u} - p_{\rho_\varnothing} B.$$

The investor always gets the same final wealth w_H, since asset managers always act in his interest. However, the fees he pays his asset manager are different for different ratings since $\Phi_{\rho_\varnothing} < \Phi_{\rho_1}$. Hence, the investor faces risk in his asset management fees. Further, the average of the fees he does pay is equal to the non-contingent fee Φ_{nc}.[24] Thus, the investor gets the same expected final monetary payoff $w_H - \Phi_{\text{nc}}$ when asset managers can offer portfolio constraints as when they cannot, except that with portfolio constraints the payoff is risky. Since the investor is risk-averse, welfare is lower with portfolio constraints. This is our next main result.

Proposition 7. (Welfare is lower with portfolio constraints.) *Welfare is lower when asset managers can offer rating-contingent portfolio constraints than when they cannot. The investor's expected payoff is strictly lower with portfolio constraints, because his payoff is riskier. In other words, ratings-based contracts prevent risk sharing, just as in the baseline model.*

Proof. The proof is in Appendix A.9. □

[24]To see this, compute the expected fee:

$$\begin{aligned}\mathbb{E}\left[\Phi\right] = \mathbb{P}\left\{\rho_1\right\}\Phi_{\rho_1} + \mathbb{P}\left\{\rho_\varnothing\right\}\Phi_{\rho_\varnothing} &= \mathbb{P}\left\{\rho_1\right\}\Big(\bar{u} - B\Big) + \mathbb{P}\left\{\rho_\varnothing\right\}\Big(\bar{u} - p_{\rho_\varnothing} B\Big)\\ &= \bar{u} - \Big(\mathbb{P}\left\{\rho_1\right\} + \mathbb{P}\left\{\rho_\varnothing\right\} p_{\rho_\varnothing}\Big)B\\ &= \bar{u} - pB \equiv \Phi_{\text{nc}}.\end{aligned}$$

Above we used the fact that the Law of Total Probability gives $p \equiv \mathbb{P}\left\{\sigma = 1\right\} = \mathbb{P}\left\{\rho_1\right\}\mathbb{P}\left\{\sigma = 1|\rho_1\right\} + \mathbb{P}\left\{\rho_\varnothing\right\}\mathbb{P}\left\{\tilde{\sigma} = 1|\rho_\varnothing\right\} = \mathbb{P}\left\{\rho_1\right\} + \mathbb{P}\left\{\rho_\varnothing\right\}\mathbb{P}\left\{\tilde{\sigma} = 1|\rho_\varnothing\right\}$, since $\mathbb{P}\left\{\tilde{\sigma} = 1|\rho_1\right\} = 1$.

6 Conclusion

In this paper we present a model of delegated asset management to understand why asset management contracts frequently depend on credit ratings. We show that asset managers contract on ratings to attract investor flows, even though contracting on ratings is not necessary to mitigate an incentive problem. Further, contracting on ratings decreases welfare by preventing risk sharing. This finding gives support to the regulatory proposal that contracting on ratings should be limited. Our equilibrium contracts share a number of features with real-world asset management contracts and our results are robust to a variety of modeling assumptions.

A Proofs

A.1 Proof of Proposition 1

We can compute the first-best contract directly by applying the first order approach[25] to the program (5):

$$\frac{\partial}{\partial \Phi}\Big(u_{\mathrm{I}}\big(w-\Phi\big)+\lambda u_{\mathrm{A}}\big(\Phi\big)\Big)=0,$$

or

$$u_{\mathrm{I}}'(w-\Phi)=\lambda u_{\mathrm{A}}'(\Phi). \tag{16}$$

By a standard result, assumption (1), that the risk tolerance is affine, implies that[26]

$$u_i'(w)=\begin{cases}(a_i+bw)^{-1/b} & \text{if } b\neq 0,\\ \\ -e^{-w/a_i} & \text{if } b=0.\end{cases} \tag{17}$$

Thus, equation (16) becomes

$$\begin{cases}\Big(a_{\mathrm{I}}+b(w-\Phi)\Big)^{-1/b}=\lambda\Big(a_{\mathrm{A}}+b\Phi\Big)^{-1/b} & \text{if } b\neq 0,\\ \\ -\exp\left(-\dfrac{w-\Phi}{a_{\mathrm{I}}}\right)=-\lambda\exp\left(-\dfrac{\Phi}{a_{\mathrm{A}}}\right) & \text{if } b=0.\end{cases} \tag{18}$$

[25] As is standard, we omit the expectation operator and maximize pointwise—if an outcome maximizes the objective at each point then it maximizes it on average.

[26] To derive this, write assumption (1) as

$$-\frac{1}{a_i+bw}=\frac{u''(w)}{u'(w)}=\Big(\log u'(w)\Big)'.$$

If $b\neq 0$, we can integrate to get $-b^{-1}\log(a_i+bw)=\log u'(w)$, which implies that $u'(w)=(a_i+bw)^{-1/b}$. If $b=0$, the condition says $a_i u''=-u'$. The solution of this differential equation is $u(w)=-e^{-w/a_i}$. (We have omitted the constants of integration; this is without loss of generality because affine transformations of a von Neumann–Morgenstern utility function are equivalent.)

This implies that

$$\Phi_{\mathrm{fb}} = \begin{cases} \dfrac{a_{\mathrm{I}} - \lambda^{-b} a_{\mathrm{A}} + bw}{b\,(1+\lambda^{-b})} & \text{if } b \neq 0, \\[2ex] \dfrac{a_{\mathrm{A}}}{a_{\mathrm{A}} + a_{\mathrm{I}}}\Big(a_{\mathrm{I}} \log \lambda + w\Big) & \text{if } b = 0, \end{cases} \tag{19}$$

which is affine in w.

A.2 Proof of Corollary 1

First, find the first-best contract using the first-order condition in equation (16),

$$u'_{\mathrm{I}}\big(w - \Phi\big) = \lambda\, u'_{\mathrm{A}}\big(\Phi\big),$$

or, for quadratic utility,

$$w - \Phi - a_{\mathrm{I}} = \lambda\big(\Phi - a_{\mathrm{A}}\big)$$

for all w. Thus the first-best contract is

$$\Phi_{\mathrm{fb}}(w) = a_{\mathrm{A}} + \frac{w - a_{\mathrm{I}} - a_{\mathrm{A}}}{1+\lambda} = A + Bw, \tag{20}$$

where

$$A = \frac{\lambda a_{\mathrm{A}} - a_{\mathrm{I}}}{1+\lambda} \quad \text{and} \quad B = \frac{1}{1+\lambda}. \tag{21}$$

Given the first-best contract, we now calculate the first-best investment in the risky security x_{fb} by computing the maximum of

$$\begin{aligned} &\mathbb{E}\left[u_{\mathrm{I}}\left(R_f + x\big(\tilde{R} - R_f\big) - \Phi_{\mathrm{fb}}\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right] \\ &\quad + \lambda \mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\mathrm{fb}}\Big(R_f + x\big(\tilde{R} - R_f\big)\Big)\right) \,\middle|\, \tilde{\sigma} = \sigma\right], \end{aligned} \tag{22}$$

over all x. That is, x_{fb} must maximize the expectation

$$-\frac{1}{2}\,\mathbb{E}\Bigg[\left(R_f + x(\tilde{R} - R_f) - A - B\left(R_f + x(\tilde{R} - R_f)\right) - a_{\mathrm{I}}\right)^2 \\ + \lambda\left(\left(A + B\left(R_f + x(\tilde{R} - R_f)\right) - a_{\mathrm{A}}\right)^2\right)\,\Big|\,\tilde{\sigma} = \sigma\Bigg]$$

over all x. Thus the first-order condition says that for optimum x_{fb}

$$\mathbb{E}\Bigg[(1-B)(\tilde{R} - R_f)\left(R_f + x_{\mathrm{fb}}(\tilde{R} - R_f) - A - B\left(R_f + x_{\mathrm{fb}}(\tilde{R} - R_f)\right) - a_{\mathrm{I}}\right) \\ + \lambda\, B(\tilde{R} - R_f)\left(A + B\left(R_f + x_{\mathrm{fb}}(\tilde{R} - R_f)\right) - a_{\mathrm{A}}\right)\,\Big|\,\tilde{\sigma} = \sigma\Bigg] = 0,$$

thus

$$x_{\mathrm{fb}} = \frac{\left(\bar{R} - R_f\right)}{\mathbb{E}\left[(\tilde{R} - R_f)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]}\left(\frac{(1-B)(A + a_{\mathrm{I}}) - \lambda\, B(A - a_{\mathrm{A}})}{(1-B)^2 + B^2\lambda} - R_f\right).$$

Substituting in for A and B from equation (21) in the numerator gives

$$(1-B)(A + a_{\mathrm{I}}) - \lambda B(A - a_{\mathrm{A}}) = \frac{\lambda\,(a_{\mathrm{A}} + a_{\mathrm{I}})}{1+\lambda}$$

and substituting in for A and B from equation (21) in the denominator gives

$$(1-B)^2 + B^2\lambda = \frac{\lambda}{1+\lambda}.$$

Therefore

$$\begin{aligned} x_{\mathrm{fb}}(\sigma) &= \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{I}} + a_{\mathrm{A}} - R_f\right)}{\mathbb{E}\left[\left(\tilde{R} - R_f\right)^2 \,\middle|\, \tilde{\sigma} = \sigma\right]} \\ &= \frac{\left(\bar{R} - R_f\right)\left(a_{\mathrm{I}} + a_{\mathrm{A}} - R_f\right)}{\sigma^2 + \left(\bar{R} - R_f\right)^2}. \end{aligned}$$

A.3 Proof of Proposition 2

To prove that the constrained-efficient outcome is the first-best outcome, we show that if the contract is the first best contract Φ_{fb}, then the incentive-compatible action is the first-best action. In other words, we show that

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\Big|\, \tilde{\sigma} = \sigma \right] \right\} \tag{23}$$

implies

$$x \in \arg\max \left\{ \mathbb{E}\left[u_{\text{I}}\big(\tilde{w} - \Phi_{\text{fb}}\big) + \lambda u_{\text{A}}\big(\Phi_{\text{fb}}\big) \,\Big|\, \tilde{\sigma} = \sigma \right] \right\}. \tag{24}$$

We begin with the asset manager's incentive problem given the contract Φ_{fb} and show through a series of manipulations that the solution coincides with that of maximizing social welfare. Incentive compatibility implies the first-order condition

$$\frac{\partial}{\partial x} \mathbb{E}\left[u_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \,\Big|\, \tilde{\sigma} = \sigma \right] = 0$$

or

$$\mathbb{E}\left[u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \Phi_{\text{fb}}'\big(\tilde{w}(x)\big) \tilde{w}'(x) \,\Big|\, \tilde{\sigma} = \sigma \right] = 0.$$

By Proposition 1 Φ_{fb}' is a constant, thus we can pass it under the expectation operator. Further, since the right-hand side above is zero, we can remove Φ_{fb}' from the equation entirely to get

$$\mathbb{E}\left[u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \tilde{w}'(x) \,\Big|\, \tilde{\sigma} = \sigma \right] = 0.$$

Now recall from equation (16) that Φ_{fb} satisfies $u_{\text{I}}'(w - \Phi) = \lambda u_{\text{A}}'(\Phi)$. Thus, we can re-write the equation above as

$$\mathbb{E}\left[u_{\text{I}}'\Big(\tilde{w}(x) - \Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big) \tilde{w}'(x) \,\Big|\, \tilde{\sigma} = \sigma \right] = 0. \tag{25}$$

Next, we manipulate this equation to recover the first-order condition for the social optimum in equation (24). To do this, we subtract the following expression from equation (25)

$$\mathbb{E}\left[\Phi_{\text{fb}}'\big(\tilde{w}(x)\big)\tilde{w}'(x)\left[u_{\text{I}}'\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)-\lambda u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\right]\,\Big|\,\tilde{\sigma}=\sigma\right]. \tag{26}$$

This expression equals zero, since, again by the definition of Φ_{fb} from equation (16), $u_{\text{I}}'(w-\Phi_{\text{fb}})-\lambda u_{\text{A}}'(\Phi_{\text{fb}})=0$. Now, factoring terms, we have

$$\begin{aligned}\mathbb{E}\left[\Big(\tilde{w}'(x)-\Phi_{\text{fb}}'\big(\tilde{w}(x)\big)\tilde{w}'(x)\Big)u_{\text{I}}'\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]+&\\ +\lambda\mathbb{E}\left[\Phi_{\text{fb}}'\big(\tilde{w}(x)\big)\tilde{w}'(x)u_{\text{A}}'\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]&=0\end{aligned}$$

or

$$\frac{\partial}{\partial x}\mathbb{E}\left[u_{\text{I}}\Big(\tilde{w}(x)-\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)+\lambda u_{\text{A}}\Big(\Phi_{\text{fb}}\big(\tilde{w}(x)\big)\Big)\,\Big|\,\tilde{\sigma}=\sigma\right]=0.$$

This is the first-order condition of the social welfare function for each σ. Since u_{I}, u_{A}, and $\tilde{w}$ are concave, the first order condition implies a global maximum, viz. the incentive compatible x is a social optimum.

A.4 Proof of Proposition 3

Suppose, in anticipation of a contradiction, an equilibrium in which the employed asset manager offers contract $\hat{\Phi}$ given rating $\hat{\rho}$ and receives strictly in excess of his reservation utility,

$$\mathbb{E}\left[u_{\text{A}}\Big(\hat{\Phi}\big(\tilde{w}\big)\Big)\,\Big|\,\tilde{\rho}=\hat{\rho}\right]>\bar{u}. \tag{27}$$

We now show that another asset manager $\hat{\text{A}}$ has a profitable deviation. In order for a contract $\hat{\Phi}_\varepsilon$ to be a profitable deviation for $\hat{\text{A}}$ it must (i) make the investor employ him given $\hat{\rho}$ and (ii) give him expected utility greater than his reservation utility $\bar{u}$ given $\hat{\rho}$. The subtlety in this proof is that $\hat{\text{A}}$'s contract determines not only the allocation of surplus, but also his action x. To circumvent the effect of changing actions on payoffs, we construct $\hat{\Phi}_\varepsilon$ to induce the asset manager to choose the same action that he would have chosen under $\hat{\Phi}$, but still to change the division of surplus. To summarize, $\hat{\Phi}_\varepsilon$ is a profitable deviation if given $\hat{\rho}$ (i) it gives the investor higher utility than does $\hat{\Phi}$,

$$\mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \hat{\Phi}_\varepsilon(\tilde{w})\right) \mid \tilde{\rho} = \hat{\rho}\right] > \mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \hat{\Phi}(\tilde{w})\right) \mid \tilde{\rho} = \hat{\rho}\right],$$

(ii) it gives the asset manager utility in excess of $\bar{u}$,

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\hat{\Phi}_\varepsilon(\tilde{w})\right) \Bigm| \tilde{\rho} = \hat{\rho}\right] > \bar{u},$$

and (iii) the set of incentive compatible actions under $\hat{\Phi}$ and $\hat{\Phi}_\varepsilon$ coincide,

$$\arg\max_x \left\{\mathbb{E}\left[u_{\mathrm{A}}\left(\hat{\Phi}_\varepsilon(\tilde{w})\right) \Bigm| \tilde{\sigma} = \sigma\right]\right\} = \arg\max_x \left\{\mathbb{E}\left[u_{\mathrm{A}}\left(\hat{\Phi}(\tilde{w})\right) \Bigm| \tilde{\sigma} = \sigma\right]\right\}.$$

One example of a contract that satisfies these three conditions is

$$\hat{\Phi}_\varepsilon(\tilde{w}) := u_{\mathrm{A}}^{-1}\left(u_{\mathrm{A}}\left(\hat{\Phi}(\tilde{w})\right) - \varepsilon\right) \tag{28}$$

given $\hat{\rho}$, so that

$$u_{\mathrm{A}}\left(\hat{\Phi}_\varepsilon\right) = u_{\mathrm{A}}\left(\hat{\Phi}\right) - \varepsilon. \tag{29}$$

Since $u_{\mathrm{I}}' > 0$, a sufficient condition for $\hat{\Phi}_\varepsilon$ to satisfy condition (i) is that

$$\tilde{w} - \hat{\Phi}_\varepsilon(\tilde{w}) > \tilde{w} - \hat{\Phi}(\tilde{w}),$$

or, substituting from equation (28),

$$\hat{\Phi}(\tilde{w}) > u_{\mathrm{A}}^{-1}\left(u_{\mathrm{A}}\left(\hat{\Phi}(\tilde{w})\right) - \varepsilon\right),$$

which is satisfied for $\varepsilon > 0$ by the inverse function theorem since $u'_{\mathrm{A}} > 0$.

Condition (ii) holds for $\varepsilon > 0$ and sufficiently small. This follows from equation (29) and inequality (27) with the continuity of u_{A}.

Finally, condition (iii) is immediate from equation (29) since affine transformations of utility do not affect choices.

Thus the investor will employ asset manager $\hat{\mathrm{A}}$ who will receive, given $\hat{\rho}$, utility greater than the utility that he would have received in the supposed equilibrium (in the supposed equilibrium he was unemployed and he was obtaining $\bar{u}$). Thus $\hat{\Phi}_{\varepsilon}$ is a profitable deviation for $\hat{\mathrm{A}}$ and Φ cannot be the contract of an asset manager employed at equilibrium given $\hat{\rho}$.

We have shown that the asset manager's expected utility given any ρ cannot exceed $\bar{u}$. To conclude the proof, note that his utility can never be strictly less than $\bar{u}$ because then his expected utility would be less than his reservation utility.

A.5 Proof of Lemma 1

The lemma follows immediately from Proposition 3.

A.6 Proof of Proposition 4

This follows directly from the solution of the constrained efficient program in Proposition 2 and the expression for the first-best contract Φ_{fb} in Proposition 1.

A.7 Proof of Proposition 5

The main step of the proof below is to show that a contract that is feasible given a fine ratings structure is also feasible given a coarse ratings structure. This follows directly from the law of iterated expectations. Since coarsening the ratings structure expands the set of feasible contracts, it can only increase the investor's objective (recall that the incentive constraints are not binding, which follows from Proposition 2). Since the asset manager always breaks even, increasing the investor's profits constitutes a Pareto improvement.

Below call $\Phi_{\lambda_{\rho_f}}$ and $\Phi_{\lambda_{\rho_c}}$ the efficient sharing rules associated with fine and coarse ratings respectively. First, the asset manager's participation constraint given $\tilde{\rho}_f$ is

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right] = \bar{u}$$

from equation (12). Now, since sigma$(\tilde{\rho}_c) \subset$ sigma$(\tilde{\rho}_f)$, use the law of iterated expectations and the condition above to observe that

$$\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right] = \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{A}}\left(\Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_f\right]\,\middle|\,\tilde{\rho}_c\right] = \mathbb{E}\left[\bar{u}\,\middle|\,\tilde{\rho}_c\right] = \bar{u}.$$

This says that $\Phi_{\lambda_{\rho_f}}$ satisfies the participation constraint given $\tilde{\rho}_c$. Since $\Phi_{\lambda_{\rho_c}}$ solves the principal-agent problem given ρ_c—viz. it maximizes the investor's utility given the asset manager's participation constraint—

$$\mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_c}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right] \geq \mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right].$$

Now we use the inequality above and we apply the law of iterated expectations again to prove that the investor is better off given the coarser ratings, namely

$$\begin{aligned}\mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_c}}(\tilde{w})\right)\right] &= \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_c}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right]\right] \\ &\geq \mathbb{E}\left[\mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\,\middle|\,\tilde{\rho}_c\right]\right] = \mathbb{E}\left[u_{\mathrm{I}}\left(\tilde{w} - \Phi_{\lambda_{\rho_f}}(\tilde{w})\right)\right].\end{aligned}$$

Since asset managers always break even and the investor is better off with coarser ratings, $\tilde{\rho}_c$ Pareto dominates $\tilde{\rho}_f$. $\square$

A.8 Proof of Proposition 5 in the Portfolio Choice Application

The proof of Proposition 5 in the portfolio choice example has two main steps. We summarize these steps briefly before giving the full proof. The first step is to show that the investor's ex ante expected utility is minus the expectation of a convex function,

$$\bar{u}\,\mathbb{E}\left[\lambda_{\tilde{\rho}}^2\right] = -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y \mid \tilde{\rho}\right]\Big)\right]$$

for (appropriately defined) $c > 0$, $f'' > 0$, and a random variable Y. The second step is to show that the expectation conditional on coarse ratings second-order stochastically dominates the expectation conditional on fine ratings,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right].$$

Whence utility is greater under coarse ratings because minus a convex function is a concave function, and, à la risk aversion, the expectation of a concave function of a stochastically dominated random variable is greater than the expectation of the function of the dominated random variable.

Before we proceed to these main steps, we derive an expression for the Lagrange multiplier λ_ρ and the investor's ex ante expected utility $\mathbb{E}\left[u_{\text{I}}\right]$. These are routine calculations, although they are somewhat lengthy.

Calculation of λ_ρ expression. We give the following expression for the Lagrange multiplier λ_ρ:

$$\left(1+\lambda_\rho\right)^2 = \frac{\left(a_{\text{P}} + a_{\text{A}} - R_f\right)^2}{2\left|\bar{u}\right|}\,\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \left(\bar{R} - R_f\right)^2}\;\middle|\;\tilde{\rho} = \rho\right]. \tag{30}$$

The expression follows from plugging in the expressions for u_{A}, Φ_ρ, and x_{fb} into the asset manager's participation constraint (12). This gives

$$
\begin{aligned}
2|\bar{u}|\big(1+\lambda_\rho\big)^2 &= \mathbb{E}\left[\left(R_f + \frac{\big(\bar{R}-R_f\big)\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\Big(\tilde{R}-R_f\Big) - a_{\mathrm{I}} - a_{\mathrm{A}}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2\,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} - 1\right)^2 \,\middle|\, \tilde{\rho}=\rho\right] \\
&= \big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2\,\left\{1 - 2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] + \right. \\
&\qquad \left. + \,\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\rho}=\rho\right]\right\}.
\end{aligned}
\tag{31}
$$

Applying the law of iterated expectations gives

$$
\begin{aligned}
&1 - \frac{2|\bar{\lambda}|\big(1+\lambda_\rho\big)^2}{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2} \\
&= 2\mathbb{E}\left[\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right] - \mathbb{E}\left[\mathbb{E}\left[\left(\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2 \,\middle|\, \tilde{\sigma}\right] \,\middle|\, \tilde{\rho}=\rho\right] \\
&= 2\mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)\mathbb{E}\left[\big(\tilde{R}-R_f\big) \,\middle|\, \tilde{\sigma}\right]}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] + \mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right]}{\Big(\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2\Big)^2} \,\middle|\, \tilde{\rho}=\rho\right].
\end{aligned}
$$

Now since

$$
\mathbb{E}\left[\big(\tilde{R}-R_f\big)^2 \,\middle|\, \tilde{\sigma}\right] = \tilde{\sigma}^2 + \big(\bar{R}-R_f\big)^2,
$$

we have

$$
\begin{aligned}
&1 - \frac{2|\bar{\lambda}|\big(1+\lambda_\rho\big)^2}{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2} \\
&= \big(\bar{R}-R_f\big)^2\left\{\mathbb{E}\left[\frac{2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right] - \mathbb{E}\left[\frac{1}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right]\right\} \\
&= \mathbb{E}\left[\frac{\big(\bar{R}-R_f\big)^2}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2} \,\middle|\, \tilde{\rho}=\rho\right].
\end{aligned}
$$

Finally, solve for $(1+\lambda_\rho)^2$ and cross multiply to recover equation (30).

Calculation of $\mathbb{E}[u_{\mathrm{I}}]$ expression. We show that the investor's ex ante expected utility can be expressed in terms of the Lagrange multiplier λ_ρ as follows:

$$\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big)\;\big|\;\tilde{\rho}=\rho\right]=\bar{u}\,\lambda_\rho^2.$$

This follows from the following string of calculations.

$$\begin{aligned}
&\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big)\;\big|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\mathbb{E}\left[\Big(a_{\mathrm{I}}-\tilde{w}+\Phi_\rho(\tilde{w})\Big)^2\;\bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\mathbb{E}\left[\left(a_{\mathrm{I}}-\tilde{w}+a_{\mathrm{A}}+\frac{\tilde{w}-a_{\mathrm{I}}-a_{\mathrm{A}}}{1+\lambda_\rho}\right)^2\;\middle|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\big(a_{\mathrm{I}}+a_{\mathrm{A}}-\tilde{w}\big)^2\;\Big|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\Big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f-x\big(\tilde{R}-R_f\big)\Big)^2\;\bigg|\;\tilde{\rho}=\rho\right]\\
&=-\frac{1}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\left(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f-\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\middle|\;\tilde{\rho}=\rho\right]\\
&=-\frac{\big(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f\big)^2}{2}\left(\frac{\lambda_\rho}{1+\lambda_\rho}\right)^2\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\middle|\;\tilde{\rho}=\rho\right].
\end{aligned}$$

Now, from equation (31) above,

$$\mathbb{E}\left[\left(1-\frac{\big(\bar{R}-R_f\big)\big(\tilde{R}-R_f\big)}{\tilde{\sigma}^2+\big(\bar{R}-R_f\big)^2}\right)^2\;\middle|\;\tilde{\rho}=\rho\right]=2|\bar{u}|\left(\frac{1+\lambda_\rho}{a_{\mathrm{I}}+a_{\mathrm{A}}-R_f}\right)^2,$$

so, finally,

$$\mathbb{E}\left[u_{\mathrm{I}}\big(\tilde{w}-\Phi_\rho\big)\;\big|\;\tilde{\rho}=\rho\right]=\bar{u}\,\lambda_\rho^2. \tag{32}$$

Main Step 1. Rewrite the investor's ex ante expected utility from the expression (32) above:

$$\begin{aligned}
\bar{u}\,\mathbb{E}\left[\lambda_{\tilde{\rho}}^2\right] &= \bar{u}\,\mathbb{E}\left[\left(\sqrt{\frac{(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2}{2|\bar{u}|}\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\,\middle|\,\tilde{\rho}\right]}-1\right)^2\right] \\
&= \frac{\bar{u}(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2}{\sqrt{2|\bar{u}|}}\mathbb{E}\left[\left[\sqrt{\mathbb{E}\left[\frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+(\bar{R}-R_f)^2}\,\middle|\,\tilde{\rho}\right]}-1\right]^2\right] \\
&= -c\,\mathbb{E}\left[f\Big(\mathbb{E}\left[Y|\tilde{\rho}\right]\Big)\right]
\end{aligned}$$

where

$$\begin{aligned}
c &:= \sqrt{|\bar{u}|/2}\,(a_{\mathrm{I}}+a_{\mathrm{A}}-R_f)^2, \\
f(z) &:= \left(\sqrt{z}-1\right)^2,
\end{aligned}$$

and

$$Y := \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2+\left(\bar{R}-R_f\right)^2}.$$

Note that $c>0$ and $f''(z)=z^{3/2}/2>0$.

Main Step 2. By definition,

$$\mathbb{E}\left[Y \mid \tilde{\rho}_c\right] \overset{\text{SOSD}}{\succ} \mathbb{E}\left[Y \mid \tilde{\rho}_f\right]$$

if there exists a random variable $\tilde{\varepsilon}$ such that

$$\mathbb{E}\left[Y \mid \tilde{\rho}_f\right] = \mathbb{E}\left[Y \mid \tilde{\rho}_c\right] + \tilde{\varepsilon}$$

and

$$\mathbb{E}\left[\tilde{\varepsilon}\,\middle|\,\mathbb{E}\left[Y \mid \tilde{\rho}_c\right]\right] = 0.$$

For $\tilde{\varepsilon} = \mathbb{E}\,[Y \mid \tilde{\rho}_f] - \mathbb{E}\,[Y \mid \tilde{\rho}_c]$ from the above, the condition is

$$\mathbb{E}\left[\mathbb{E}\,[Y \mid \tilde{\rho}_f] - \mathbb{E}\,[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}\,[Y \mid \tilde{\rho}_c]\right] = 0$$

or

$$\mathbb{E}\left[\mathbb{E}\,[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}\,[Y \mid \tilde{\rho}_c]\right] = \mathbb{E}\,[Y \mid \tilde{\rho}_c].$$

Given the assumption $\text{sigma}(\tilde{\rho}_c) \subset \text{sigma}(\tilde{\rho}_f)$ and since conditioning destroys information—$\text{sigma}\big(\mathbb{E}\,[Y \mid \tilde{\rho}_c]\big) \subset \text{sigma}(\tilde{\rho}_c)$—apply the law of iterated expectations firstly to add and then to delete conditioning information to calculate that

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\,[Y \mid \tilde{\rho}_f] \,\middle|\, \mathbb{E}\,[Y \mid \tilde{\rho}_c]\right] &= \mathbb{E}\left[\mathbb{E}\left[\mathbb{E}\,[Y \mid \tilde{\rho}_f] \,\middle|\, \tilde{\rho}_c\right] \,\middle|\, \mathbb{E}\,[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[\mathbb{E}\,[Y \mid \tilde{\rho}_c] \,\middle|\, \mathbb{E}\,[Y \mid \tilde{\rho}_c]\right] \\
&= \mathbb{E}\left[Y \,\middle|\, \tilde{\rho}_c\right],
\end{aligned}$$

as desired.

A.9 Proof of Proposition 7

Most of the argument is in the text. However, we have not shown that the investor does indeed choose the non-contingent contract $\Phi_{\rho_\varnothing}$ when $\tilde{\rho} = \rho_\varnothing$. We show this now.

Suppose that $\tilde{\rho} = \rho_\varnothing$. If the investor chooses the non-contingent contract $\Phi_{\rho_\varnothing}$ the asset manager always takes the right action. The investor's monetary payoff is deterministic and equal to

$$w_H - \Phi_{\rho_\varnothing} = w_H - \bar{u} + p_{\rho_\varnothing} B. \tag{33}$$

If the investor chooses the contingent contract Φ_{ρ_1} the asset manager takes the right action when $\tilde{\sigma} = 0$ and takes the wrong action when $\tilde{\sigma} = 1$. The investor's monetary payoff is thus risky with expected value equal to

$$\mathbb{P}\left\{\tilde{\sigma}=0 \,|\, \rho_\varnothing\right\} w_H + \mathbb{P}\left\{\tilde{\sigma}=1 \,|\, \rho_\varnothing\right\} w_L - \Phi_{\rho_1} = \left(1-p_{\rho_\varnothing}\right) w_H + p_{\rho_\varnothing} w_L - \bar{u} + B.$$

This expression is less than the payoff (33) from choosing $\Phi_{\rho_\varnothing}$ as long as

$$p_{\rho_\varnothing}\left(w_H - w_L\right) > \left(1-p_{\rho_\varnothing}\right) B,$$

which is satisfied by assumption (14). Thus, the payoff from choosing $\Phi_{\rho_\varnothing}$ has less risk and higher expected value than the payoff from choosing Φ_{ρ_1}, so the investor always chooses the non-contingent contract when $\tilde{\rho}=\rho_\varnothing$, as desired. □

B Table of Notations

	Indices
I	index indicating the investor
A	index indicating an asset manager
	Utility Parameters
u_i	utility function of player $i \in \{\mathrm{I}, \mathrm{A}\}$
a_i	risk-aversion parameter of player $i \in \{\mathrm{I}, \mathrm{A}\}$ (see equation (1))
b	common utility parameter
	Signals/Information Structure
σ	asset managers' private signal
ρ	rating
	Contracts, Actions, and Payoffs
Φ	contract an asset manager offers the investor
Φ_{fb}	first-best contract (Subsection 3.1)
Φ'	$\partial\Phi/\partial w$, the derivative of Φ with respect to the first argument
Φ_ρ	the contract Φ for a given rating[27]
x	asset manager's action or "portfolio weight"
w	final wealth
	Application to Portfolio Choice
R_f	gross risk-free rate
$\tilde{R}$	gross rate of return on risky asset
$\bar{R}$	expected gross rate of return on the risky asset
	Alternative Model in Section 5
Φ	the flat fee paid to the asset manager
p	probability that $\tilde{\sigma} = 1$
$p_{\rho_\varnothing}$	probability that $\sigma = 1$ given $\rho = \rho_\varnothing$
B	asset manager's private benefit from choosing $x = 1$
	Other Quantities and Notations
λ	social planner's welfare weight on the asset manager (Subsection 3.1)
λ_ρ	Lagrange multiplier on the asset manager's participation constraint (Subsection 4.2)
sigma$(\cdot)$	the sigma-algebra generated by a random variable

[27] This notation serves to emphasize that Φ actually depends on ρ, even though $\Phi = \Phi(w, x, \rho)$ is always a function of ρ.

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