

# MONEY RUNS\*

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## Abstract

We present a banking model in which bank debt circulates in decentralized secondary markets, like banknotes did in the nineteenth century and repos do today. We find that bank debt is susceptible to runs because secondary-market liquidity is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a “money run.” Even though demandable debt exposes banks to costly runs, banks still choose to issue it: the option to redeem on demand increases secondary-market debt prices and hence increases primary-market debt capacity—i.e. demandability and tradeability are complements, unlike in existing models.

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## 1 Introduction

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Bank debt was a major form of money in the nineteenth century United States. To get beer from the barman, you would exchange banknotes over the counter. Banknotes were redeemable on demand and sudden redemptions—bank runs—were common. Bank debt remains a major form of money today. To get liquidity from a financial counterparty, you exchange repos in an over-the-counter (OTC) market. Repos are effectively redeemable on demand and sudden redemptions—repo runs—were a salient event of the 2008–2009 financial crisis.<sup>1,2</sup> In summary, when you hold bank debt, you can get liquidity either by trading OTC or, alternatively, by demanding redemption from the issuing bank. But demanding redemption comes with the risk of a run. Why would you run on a bank rather than trade its debt in the market? In other words, why is bank debt susceptible to costly runs, even though it is tradeable? Moreover, why do banks choose to borrow via demandable debt, even though it exposes them to costly runs?

To give a new perspective on these questions, we focus on how banks create money by issuing liabilities that circulate in OTC markets, like banknotes did in the nineteenth-century and repos do today. In the model, bank debt is susceptible to runs because liquidity in the OTC market is fragile, and subject to sudden, self-fulfilling dry-ups.

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<sup>1</sup>Gorton and Metrick (2009, 2012) and Krishnamurthy, Nagel, and Orlov (2014) provide descriptions of repo runs.

<sup>2</sup>Gorton (2012b) argues that it remains a theoretical challenge to understand how these runs arise and how they affect the design of bank liabilities that circulate as money. He suggests that, because banknotes and repos are backed by collateral, there is no “common pool problem” inducing depositors to race to withdraw first in the event of a crisis:

In the U.S. under state free banking laws banks were required to back their notes with state bonds. In the case of a bank failure—an inability to honor requests for cash from noteholders—the state bonds would be sold (by the state government) and the note holders paid off pro rata. Note holders were paid off pro rata, so there was no common pool problem. Yet, there was a run on banks (banknotes and deposits) during the Panic of 1857 (p. 15).

And he goes on to say:

Generating such [a run] event in a model seems harder when...the form of money [is such that] each “depositor” receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities—private money (p. 2).

We generate such runs on bank debt in a model in which banks optimally design securities that circulate in secondary markets. These runs occur because strategic considerations about coordinating with other agents do arise in the secondary market. (Other papers, such as Martin, Skeie, and von Thadden (2014a, 2014b) and Kuong (2015) study other mechanisms by which runs on backed debt can occur.)

When debt fails to circulate it is redeemed on demand in a bank run, or “money run.” Such runs were common in the nineteenth century US, when depositors ran on banks after “the bank note that passed freely yesterday was rejected this morning” (Treasury Secretary Howell Cobb (1858), quoted in Gorton (2012a) p. 36). Even though demandable debt exposes banks to costly runs, banks still choose to issue it. In our model, this is because it increases banks’ debt capacity: the option to redeem on demand props up the price of debt in the *secondary market*. In other words, we find that demandability and tradeability are complements. This contrasts with Jacklin’s (1987) argument that, roughly, you do not need the option to redeem debt on demand if you can just trade it in the secondary market. In our model, on the contrary, you do: the option to redeem debt on demand increases the price at which you trade it in the secondary market. This benefits the issuing bank by allowing it to borrow more. As a result, financial fragility is a necessary evil—it is the cost of the debt capacity afforded by demandable debt. Overall, our model exposes a new type of run and a new rationale for demandable debt, both of which are based on the circulation of bank liabilities in the secondary market.

**Model preview.** In the model, a borrower  $B$  has an investment opportunity and needs to borrow from a creditor  $C_0$  to fund it. The model is based on two key assumptions. First, there is a horizon mismatch, similar to that in Diamond and Dybvig (1983):  $C_0$  may be hit by a liquidity shock before  $B$ ’s investment pays off. This mismatch implies that  $B$  does maturity transformation, and therefore resembles a bank. Second,  $B$ ’s debt is traded in an OTC market, similar to that in Trejos and Wright (1995) and Duffie, Gârleanu, and Pedersen (2005): if  $C_0$  is hit by a liquidity shock before  $B$ ’s investment pays off,  $C_0$  can match with a counterparty  $C_1$  and bargain bilaterally to trade  $B$ ’s debt. Likewise,  $C_1$  may be hit by a liquidity shock before  $B$ ’s investment pays off, in which case it can match with a counterparty  $C_2$  and bargain bilaterally to trade  $B$ ’s debt, and so on. If  $B$ ’s debt is demandable, then a creditor may redeem it before the investment pays off, forcing  $B$  to liquidate inefficiently.

**Results preview.** Our first main result is that  $B$ ’s debt capacity is highest if it issues tradeable, demandable debt, which we refer to as a “banknote.” In particular, as long as the horizon mismatch is sufficiently severe,  $B$  cannot fund its investment with non-tradeable debt (e.g. a bank loan), even if it is demandable, or with non-demandable debt (e.g. a bond), even if it is tradeable. To see why this is, consider  $C_0$ ’s decision whether or not to lend to  $B$ .  $C_0$  knows that he may be hit by a liquidity shock before  $B$ ’s investment pays off, in which case  $C_0$  liquidates  $B$ ’s debt, either by redeeming on demand or by trading in the OTC market. If  $B$ ’s debt is not tradeable (but is demandable), then  $C_0$  must redeem on demand, forcing  $B$  into inefficient liquidation and recovering less than his initial investment. If the horizon mismatch is severe, then this loss from early redemption is so likely that  $C_0$  is unwilling to lend in the first place.

In contrast, if B’s debt is tradeable (but is not demandable), then  $C_0$  can avoid early redemption by trading with  $C_1$  in the OTC market. However,  $C_0$ ’s liquidity shock puts him in a weak bargaining position with  $C_1$ :  $C_0$  has a low outside option because he has no way to get liquidity if trade fails. As a result, he sells B’s debt at a discounted price, recovering less than his initial investment. If the horizon mismatch is severe, then this loss from selling at a discount is so likely that  $C_0$  is unwilling to lend in the first place. But if B’s debt is demandable as well as tradeable, then debt does not trade at such a high discount in the secondary market. This is because demandability improves  $C_0$ ’s bargaining position with  $C_1$ . It increases his outside option, since he can redeem on demand when trade fails. As a result,  $C_0$  can trade B’s debt at a high price following a liquidity shock. Thus,  $C_0$  is insured against liquidity shocks, making him willing to fund B’s investment. In contrast to existing models of demandable debt, our model suggests that demandability complements tradeability—just the option to redeem on demand props up the resale price of debt in secondary markets even if the debt is never actually redeemed on demand in any state of the world.

Our second main result is that banknotes are susceptible to a new kind of bank run, which results directly from the dry-up of secondary-market liquidity. Specifically, a sudden (but rational) change in beliefs can cause counterparties to stop trading in the secondary market, leading the creditor to redeem on demand and forcing B into inefficient liquidation. Observe that this run occurs even though B has only a single creditor—there is no static coordination problem in which multiple creditors race to be the first to withdraw from an issuing bank as in Diamond and Dybvig (1983); rather, there is a dynamic coordination problem in the secondary market in which a counterparty does not accept B’s debt today because he is worried that his future counterparty will not accept B’s debt tomorrow. Due to this self-fulfilling liquidity dry-up, B’s creditor is suddenly unable to trade when he is hit by a liquidity shock and, thus, he must demand redemption from B. We refer to this run as a “money run” because it is the result of the failure of B’s debt to function as a liquid money in the secondary market.

We also show that if the horizon mismatch is not too severe, B may choose to borrow via a bond, i.e. tradeable, non-demandable debt, instead of a banknote. The reason is not only that bond liquidity dry-ups do not lead to bank runs and inefficient liquidation, but also that bond liquidity dry-ups are relatively “unlikely.” This is because bonds trade at a larger discount than banknotes, making counterparties more willing to trade in the secondary market (Subsection 3.8).

Finally, we explore our model’s implications for asset choice and intermediation. We show that if B can choose its investment, its choice may be distorted toward high-liquidation-value investments, which facilitate its issuing demandable debt (Subsection 3.9). Further, this need to issue demandable debt can give rise to intermediation. We

show that if B has invested in assets with high liquidation value, then B may emerge as an intermediary between B' and  $C_0$  (Subsection 3.10). The reason is that, unlike B', B can issue demandable debt, since it can use its liquid assets to back its debt to  $C_0$ . In other words, B becomes the banker only because it can create circulating liabilities—private money.

**Policy.** Our analysis suggests that financial fragility may be a necessary evil given secondary-market trading frictions—money runs are the cost of the debt capacity afforded by demandable debt. However, decreasing secondary-market trading frictions can make banks less reliant on demandable debt, decreasing the likelihood of runs. Thus, we suggest that to improve bank stability, a policy maker may be better off decreasing secondary-market frictions than regulating banks directly. In contemporary markets, we think that centralized exchanges and clearing houses for bank bonds, like those for stocks, could decrease trading frictions in a way that decreases banks' reliance on overnight (effectively demandable) debt.

Unlike in Diamond and Dybvig's (1983) model of bank runs, in which suspension of convertibility restores efficiency, in our model suspension of convertibility may have an adverse effect. Since it prevents creditors from redeeming on demand to meet their liquidity needs, it leads to lower secondary-market debt prices and, hence, to constrained bank borrowing and inefficient investment.

**Empirical content.** As mentioned above, our model is motivated by empirical observations about financial fragility and circulating bank debt, such as banknotes and repos. In particular, our model offers an explanation of the following facts: (i) runs on bank debt are relatively common, even when the debt is backed by collateral; (ii) runs are often precipitated by the failure of debt to circulate in secondary markets; and (iii) banks choose to borrow via demandable debt even though it exposes them to costly runs.

Our model also casts light on several other stylized facts. (i) Demandable bank instruments, such as banknotes, deposits, and repos, are more likely to serve as media of exchange than other negotiable instruments, such as bonds and shares. In the model, this is because the option to redeem on demand props up the secondary-market price of bank debt. Thus, if you hold a variety of instruments, you prefer to use demandable instruments to raise liquidity in the secondary market and to hold long-term instruments till maturity. (ii) Our model casts light on why bank debt is more likely to be demandable than corporate debt. Banks, almost by definition, have a horizon mismatch between their assets and liabilities—they perform maturity transformation. In the model, this horizon mismatch prevents you from borrowing via other instruments. Corporates are less likely to suffer from the horizon mismatch, and are therefore more likely to fund themselves with bonds or bank loans, which do not expose them to costly

runs/liquidation. (iii) Our model casts light on why nineteenth-century banknotes traded at a greater discount in markets farther away from the issuing bank: distance from the issuer made the notes harder to redeem on demand, weakening note holders' bargaining positions in the secondary market and decreasing the price of banknotes (see Gorton (1996)). (iv) Our model generates runs even with a single depositor, consistent with the fact that many runs are not market-wide, but rather occur in isolation. Indeed, Krishnamurthy, Nagel, and Orlov (2014) find that repo runs occurred in relative isolation during the financial crisis.

**Application to repos.** Formally, repos are collateralized bilateral contracts, not circulating negotiable instruments like the banknotes in our model. However, repos share the key features of our banknotes: in addition to being exchanged OTC, they are effectively demandable and tradeable. They are effectively demandable because they are continuously rolled over (not settled and reopened daily). Thus, not rolling over a repo position is effectively redeeming on demand. They are effectively tradeable because repo collateral is constantly rehypothecated<sup>3</sup> (see Singh and Aitken (2010) and Singh (2010)). Thus, repo creditors use repos to get liquidity from third parties in the event of a liquidity shock, as creditors use tradeable debt to get liquidity in our model.

**Related literature.** We make three main contributions to the literature.

First, we offer a new rationale for demandable debt. This adds to the literature in two ways. (i) It complements the literature that shows how demandability can mitigate moral hazard problems (Calomiris and Kahn (1991) and Diamond and Rajan (2001a, 2001b)).<sup>4</sup> In particular, we show how demandability can facilitate secondary-market trade in “private money.” Thus, our model connects two of the main features of bank liabilities: they both circulate as money and are redeemable on demand. (ii) It provides a counterpoint to the literature that suggests that tradeability can substitute for demandability. Notably, Jacklin (1987) shows that, in Diamond and Dybvig's (1983) environment, you do not need to redeem debt on demand if you can just trade it in the secondary market.<sup>5</sup> We show that if bank debt is traded in an OTC market, like

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<sup>3</sup>So whereas the repo contract itself technically does not circulate, the repo collateral does. See Lee (2015) for a model focusing on collateral circulation.

<sup>4</sup>In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating bank notes informally, saying that

deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

<sup>5</sup>Other papers show that there may still be a role for demandability if tradeability is limited (Allen and Gale (2004), Antinolfi and Prasad (2008), Diamond (1997), and von Thadden (1999)). In these models, banks issue demandable debt *in spite of* trade in secondary markets, e.g. to overcome trading frictions, such as limited market participation. In our model, banks issue demandable debt *because of* trade in secondary

banknotes, deposits, and repos are, then demandability serves another purpose: it improves sellers' bargaining positions and thus increases the secondary-market price of bank debt. Thus, demandability complements tradeability in our model.

Second, we uncover a new kind of bank run. By connecting the fragility of money to the fragility of banks, this adds both to the literature on coordination-based bank-run models following Diamond and Dybvig (1983) and to the literature on search-based money models following Kiyotaki and Wright (1989, 1993). In these money models, monetary exchange is fragile since trade is self-fulfilling. Similarly, in the bank run models, bank deposits are fragile since withdrawals are self-fulfilling. To the best of our knowledge, we are the first to show that such bank fragility follows immediately from such monetary fragility<sup>6</sup> and, hence, coordination-based bank runs can occur even with a single depositor—i.e. without multiple depositors racing to withdraw from a common pool of assets as in Diamond and Dybvig (1983).<sup>7</sup> This helps to explain how runs can occur on collateral-backed debt, complementing the existing literature (see footnote 2).

Third, we study security design when securities are traded OTC in the secondary market. This adds to the literature in three ways. (i) It complements the search-based money literature which analyzes which type of asset is the socially optimal medium of exchange for trade in the secondary market (e.g., Kiyotaki and Wright (1989) and Burdett, Trejos, and Wright (2001)). We analyze which type of contract is the privately optimal circulating instrument for funding in the primary market.<sup>8</sup> (ii) It extends results in the literature on corporate bonds that suggests short-maturity bonds may have high resale prices in the secondary market (Bruche and Segura (2016) and He and Milbradt (2014)). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts the option to redeem on demand provides a way to prop up secondary-market prices that does not

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markets—the option to redeem on demand improves the terms of trade in the secondary market.

<sup>6</sup>A number of papers study bank money creation independently of financial fragility (e.g., Donaldson, Piacentino, and Thakor (2017), Gu, Mattesini, Monnet, and Wright (2013), and Kiyotaki and Moore (2001a)) and some others embed Diamond–Dybvig runs in economies with private money (e.g., Champ, Smith, and Williamson (1996) and Sanches (2015)). Relatedly, Sanches (2016) argues that banks' inability to commit to redeem deposits can make private money unstable.

<sup>7</sup>Our focus on runs that result from dynamic coordination failures among counterparties in the secondary market complements models that focus on runs that result from dynamic coordination failures among depositors in the primary market (the dynamic analog of Diamond–Dybvig-type runs), such as He and Xiong (2012).

<sup>8</sup>A related line of research focuses on information, rather than OTC trading frictions, in secondary-market trade (Gorton and Pennacchi (1990), Dang, Gorton, and Hölmstrom (2015a, 2015b), Dang, Gorton, Hölmstrom, and Ordoñez (forthcoming), Gorton and Ordoñez (2014), Jacklin (1989), and Vanasco (2016)). This literature suggests that information frictions in the secondary market lead banks to do risk transformation; analogously, we suggest that OTC trading frictions in the secondary market lead banks to do liquidity transformation.

require rolling over maturing debt.<sup>9,10</sup> (iii) It provides a counterpoint to the literature that suggests that security design may prevent bank runs (e.g. Andolfatto, Nosal, and Sultanum (forthcoming), Green and Lin (2003), and Peck and Shell (2003)). This literature suggests that if the space of securities is rich enough, then bank runs do not arise in Diamond and Dybvig’s (1983) environment. Our analysis suggests that the security designs proposed in this literature may not prevent all kinds of bank runs. This is because, in our environment, it is exactly the possibility of a run, i.e. the option to redeem on demand, that props up the secondary-market price of the banknote and thus makes it the optimal funding instrument.

**Layout.** In Section 2, we present the model. In Section 3, we present the main results. Section 4 is the conclusion. The Appendix contains all proofs and a table of notations.

## 2 Model

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In this section, we present the model.

### 2.1 Players, Dates, and Technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite,  $t \in \{0, 1, \dots\}$ .

There are two types of players, a single penniless borrower B and infinitely many deep-pocketed creditors  $C_0, C_1, \dots$ . Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs  $c$  at Date 0 and pays off  $y > c$  at a random time in the future, which arrives with intensity  $\rho$ . Thus, the investment has NPV  $= y - c > 0$  and expected horizon  $1/\rho$ . B may also liquidate the investment before it pays off; the liquidation value is  $\ell < c/2$ .

B can fund its project by borrowing from a creditor. However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to con-

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<sup>9</sup>In an extension, Bruche and Segura (2016) do consider a version of puttable debt. However, they effectively assume it is not tradeable, which shuts down the interaction of demandability and tradeability that is critical to our results.

<sup>10</sup>It may be worth emphasizing that we do not consider the rollover strategy in which B borrows via one-period contracts at each date. This rollover of short-term debt is different from demandable debt—it is necessarily redeemed at each date, whereas demandable debt is redeemed only if debtholders exercise their option to demand. This is the difference between, e.g., commercial paper, which has a fixed maturity, and repos, which typically have open maturity. We abstract from such short-term debts to keep our analysis simple and distinguish it from the rollover-risk literature. That said, as long as rolling over has the same cost as secondary market trade, it is less desirable than issuing demandable debt in our environment. The reason is that the cost would be paid each date no matter what, instead of only when debtholders want to trade.



sume before B’s investment pays off. Specifically, creditors consume only if they suffer “liquidity shocks,” which arrive at independent random times with intensity  $\theta$  (after which they die). In other words, a creditor’s expected “liquidity horizon” is  $1/\theta$ .

## 2.2 Borrowing Instruments

At Date 0, B borrows the investment cost  $c$  from its initial creditor  $C_0$  and negotiates a repayment  $R \leq y$  to make when the investment pays off. We refer to this promised repayment as B’s “debt.” (However, it can also represent an equity claim; debt and equity have equivalent payoffs, since the terminal payoff  $y$  is deterministic.)

In addition to the repayment  $R$ , two other characteristics define B’s debt: *tradeability* and *demandability*. If B’s debt is non-tradeable, then B must repay  $C_0$ .<sup>11</sup> In contrast, if B’s debt is tradeable, creditors can exchange the debt among themselves—the debtholder at Date  $t$ , denoted  $H_t$ , may not be the initial creditor  $C_0$ —and B must repay whichever creditor  $H_t$  holds its debt. If B’s debt is not demandable, i.e. it is “long-term,” then B repays only when its investment pays off. In contrast, if B’s debt is demandable, then the debtholder can demand repayment at any date. In this case, if B’s investment has not paid off, B liquidates its investment and repays the liquidation value  $\ell$ .

In summary, B borrows via one of four types of debt instrument: (i) non-tradeable long-term debt, which we refer to as a “loan”; (ii) non-tradeable demandable debt, which we refer to as a “puttable loan”; (iii) tradeable long-term debt, which we refer to as a “bond”; or (iv) tradeable demandable debt, which we refer to as a “banknote,” although it also resembles a bank deposit or a repo. These instruments are summarized in Figure 1. These instruments are effectively all of the feasible Markovian instruments, i.e. contracts that can depend on the state of B’s investment at Date  $t$ , but not on the date itself, and do not violate B’s limited-liability constraint.<sup>12</sup>

We let  $v_t$  denote the Date- $t$  value of B’s debt to a creditor not hit by a liquidity

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<sup>11</sup>In the Diamond and Dybvig (1983) environment, Jacklin (1987) shows that the only disadvantage of a tradeable instrument relative to demandable debt is that it must be tradeable in a market at the interim date, and that players’ initial investments may be distorted in anticipation of trading in this market (see also Allen and Gale (2004) and Farhi, Golosov, and Tsyvinski (2009)). Kiyotaki and Moore (2001a, 2001b, 2002, 2005) also differentiate between tradeable and non-tradeable debt, taking tradeability as exogenous. Donaldson and Micheler (2016) analyze a model in which tradeability is chosen by the borrower, as it is here.

<sup>12</sup>Here we are making two implicit assumptions that may be worth highlighting. (i) We have implicitly ruled out contracts that depend on creditors’ liquidity shocks. This is an important assumption to generate a reason for a secondary market—if liquidity shocks were contractable, then there would be no role for retrade since the first best could be implemented with contingent contracts à la Arrow–Debreu. In other words, we assume that the Markov state is the state of B’s investment only. (ii) We have implicitly ruled out contracts in which B promises a repayment less than the liquidation value  $\ell$  in the event that debt is demanded early. This is just for simplicity—it helps to streamline the analysis but does not substantively affect the equilibrium.

shock.

FIGURE 1: DEBT INSTRUMENTS

	not demandable	demandable
non-tradeable	“loan”	“puttable loan”
tradeable	“bond”	“banknote” (deposits or repos)

### 2.3 Secondary Debt Market: Entry, Bargaining, and Settlement

If B has borrowed via tradeable debt, then creditors can trade it bilaterally in an OTC market. At each Date  $t$ ,  $C_t$  is the (potential) counterparty with whom the debtholder  $H_t$  may trade B’s debt.  $C_t$  can pay an entry cost<sup>13</sup>  $k$  to be matched with the debtholder  $H_t$ . If matched,  $C_t$  and  $H_t$  Nash bargain to determine the price  $p_t$  (all Nash bargaining is symmetric).<sup>14</sup> If they agree on a price, then trade is settled:  $C_t$  becomes the debtholder in exchange for  $p_t$  units of the good. Otherwise,  $H_t$  retains the debt. If the debt is demandable,  $H_t$  can demand redemption from B or he can remain the debtholder at Date  $t + 1$ . This sequence of entry, bargaining, and settlement is illustrated in Figure 2.

We let  $\sigma_t$  denote  $C_t$ ’s mixed strategy if  $H_t$  is hit by a liquidity shock, so  $\sigma_t = 1$  means that  $C_t$  enters for sure and  $\sigma_t = 0$  means that  $C_t$  does not enter. Thus,  $\sigma_t$  also represents the probability that  $H_t$  finds a counterparty when hit by a liquidity shock. Observe that we restrict attention to  $C_t$ ’s strategy given  $H_t$  is hit by a liquidity shock without loss of generality.<sup>15</sup>

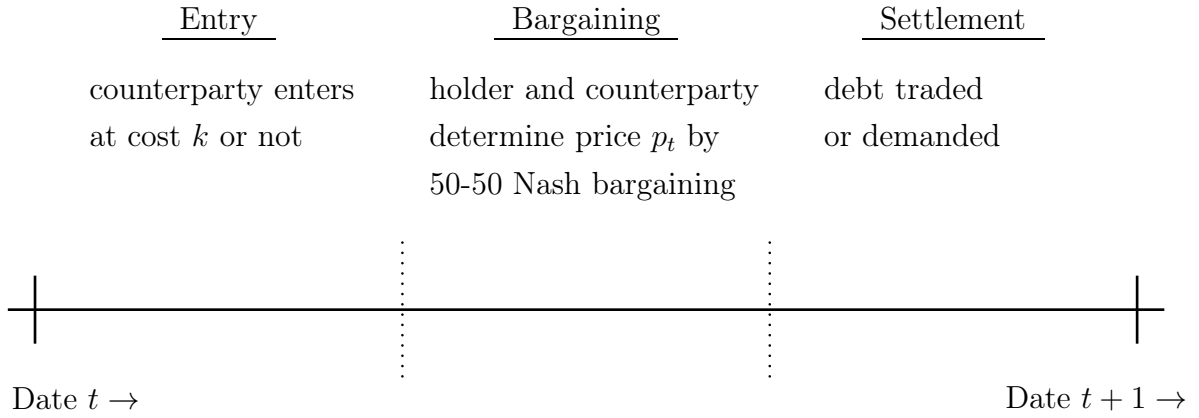
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<sup>13</sup>Historically, this “entry” cost could represent the physical cost of coming to market or, alternatively, of acquiring the expertise/technology to check for counterfeit instruments. Today, it could represent the cost of setting up a trading desk to participate in a specific market (e.g. the repo market) or, alternatively, of establishing the legal infrastructure to handle certain instruments (e.g. the GMRA master agreement for repos). More generally, it could represent any cost of searching for a counterparty as in the search money literature, of trading/transacting as in the finance literature, or of posting a vacancy, as in the labor literature. All that matters for our results is that  $C_t$  bears some fixed cost to get B’s debt from  $H_t$ . Further, our main results are only slightly modified if this entry cost is zero but the debtholder must pay a per-period holding cost  $\delta$  as in Duffie, Gârleanu, and Pedersen (2005) (setting  $\delta = (\rho + (1 - \rho)\theta)k$  gives the same ex ante surplus to  $H_t$  and  $C_t$ ).

<sup>14</sup>To economize on notation, we assume that all bargaining is “symmetric” or “fifty-fifty” Nash bargaining as in Nash (1950). With appropriate modifications of conditions, our results also hold if we use the generalized Nash bargaining outcome, in which  $H_t$  and  $C_t$  split the surplus in arbitrary fixed proportions. It is important, however, that  $H_t$  does not have all the bargaining power, since, in this case, his outside option does not affect the division of surplus. This undoes the effect of demandability.

<sup>15</sup>The reason that this is without loss of generality is that  $C_t$  would never enter if  $H_t$  were not hit by a liquidity shock: if  $H_t$  is not hit by a liquidity shock,  $H_t$  and  $C_t$  are identical and there are no gains from trade, so it is never worth it to pay the cost  $k$  for the opportunity to trade.

FIGURE 2: SECONDARY-MARKET TRADE



## 2.4 Timeline

First, B chooses a debt instrument—a loan, a puttable loan, a bond, or a banknote, as described in Subsection 2.2 above. Next, B and the initial creditor  $C_0$  negotiate the repayment  $R$  or fail to reach an agreement. If B and  $C_0$  agree on  $R$ , then  $C_0$  becomes the initial debtholder. Depending on the instrument, the debtholder may redeem on demand or may trade in the secondary market, as described in Subsection 2.3 above. Formally, the extensive form is as follows.

<u>Date 0</u>	B chooses a debt instrument
	B is matched with $C_0$ ; B and $C_0$ Nash bargain to determine the repayment $R$
	If B and $C_0$ agree on $R$ , then B invests $c$ . $C_0$ is the initial debtholder, $H_1 = C_0$
<u>Date <math>t &gt; 0</math></u>	<u>If B's investment pays off</u>
	B repays $R$ to $H_t$ and B consumes $y - R$
	<u>If B's investment does not pay off:</u> there is entry, bargaining, and settlement as described in Subsection 2.3
	If there is trade, $C_t$ becomes the new debtholder, $H_{t+1} = C_t$
	If there is no trade, $H_t$ either holds the debt, $H_{t+1} = H_t$ , or redeems on demand, in which case B repays $\ell$ to $H_t$ and B consumes zero

## 2.5 Equilibrium

The solution concept is subgame perfect equilibrium. An equilibrium constitutes (i) the instrument and associated repayment  $R$ , (ii) the price of debt in the secondary market

$p_t$  at each date, and (iii) the entry strategy  $\sigma_t$  of the potential counterparty  $C_t$ <sup>16</sup> such that B’s choice of instrument and  $C_t$ ’s choice to enter are sequentially rational,  $R$  and  $p_t$  are determined by Nash bargaining, and each player’s beliefs are consistent with other players’ strategies and the outcomes of Nash bargaining.

We focus on pure, stationary equilibria, i.e.  $\sigma_t \equiv \sigma \in \{0, 1\}$  and  $p_t \equiv p$ .

## 2.6 Assumption: Horizon Mismatch

We assume that parameters are such that there is a relatively severe horizon mismatch between B’s investment and creditors’ liquidity needs, as in Diamond and Dybvig (1983). In our infinite-horizon environment, this implies that the expected investment horizon  $1/\rho$  must be sufficiently large relative to the expected liquidity horizon  $1/\theta$ . Specifically, we assume that the following condition holds:

$$\frac{1}{\rho} > \frac{1}{\theta} \frac{2(y - c)}{c(1 - \rho)}. \quad (\star)$$

This assumption implies that B intermediates between short-horizon creditors and a long-horizon investment. This implies B resembles a bank, since, almost by definition, banks transform maturity to meet the needs of short-term depositors and long-term borrowers. It also implies that B’s debt is a kind of inside money, since creditors generally will not hold it for its entire maturity; rather they will hold it for a short period of time and then use it to get liquidity from another creditor—as Kiyotaki and Moore (2001a) put it, “[w]henver paper circulates as a means of short-term saving (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value” (p. 1).

This assumption helps to present our most interesting/novel results in a simple way. We do not need it to characterize the equilibrium. Indeed, we relax it in Subsection 3.8.

## 3 Results

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In this section, we present our main results.

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<sup>16</sup>Formally, we could also include the debtholder’s choice whether to redeem on demand. We omit this, however, and just assume that the debtholder redeems on demand whenever he is hit by a liquidity shock and does not trade the debt. This assumption is just for simplicity; it does not affect the equilibrium.

### 3.1 Borrowing Constraint

We begin the analysis with a necessary condition for B to be able to borrow enough to fund its investment.

LEMMA 1. *For a given instrument, B can borrow and invest if and only if the Date-0 value of its debt exceeds the cost of investment, i.e. if and only if*

$$v_0 \geq c \tag{$}$$

for some repayment  $R \leq y$ .

We make use of this borrowing constraint repeatedly below when we consider each type of debt instrument in turn and ask whether B can borrow enough to undertake its investment.

### 3.2 Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date  $t$ , the value  $v_t$  of the loan can be written recursively:

$$v_t = \rho R + (1 - \rho)(1 - \theta)v_{t+1}. \tag{1}$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. Since the loan is neither tradeable nor demandable,  $H_t$  gets zero. With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains B's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.<sup>17</sup> By stationarity ( $v_t = v_{t+1} \equiv v$ ), equation (1) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta}. \tag{2}$$

Given the horizon mismatch assumption ( $\star$ ), the expression above is always less than the cost of investment  $c$ . Thus, if B borrows via a loan, B cannot satisfy its borrowing constraint ( $\$$ ) and therefore cannot invest.

LEMMA 2. *If B borrows via a loan, it cannot fund its investment.*

B cannot borrow from  $C_0$  via a loan even though its project has positive NPV. This is because, given the horizon mismatch,  $C_0$  is likely to be hit by a liquidity shock and

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<sup>17</sup>Formally, the value of holding B's debt is the Date- $t$  *expected* value of B's debt at Date  $t + 1$ , i.e. we should write  $\mathbb{E}_t[v_{t+1}]$  instead of  $v_{t+1}$ . For now, we focus on deterministic equilibria. Thus, this difference is immaterial and we omit the expectation operator for simplicity. (In Subsection 3.7, we do keep track of the expectation operator.)

need to consume before the project pays off, in which case  $C_0$  gets zero, since the loan is neither tradeable nor demandable.

### 3.3 Puttable Loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date  $t$ , the value  $v_t$  of the puttable loan can be written recursively:

$$v_t = \rho R + (1 - \rho) \left( \theta \ell + (1 - \theta) v_{t+1} \right). \quad (3)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. Since the loan is demandable, but not tradeable,  $H_t$  redeems on demand and gets  $\ell$ . With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains B's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.

By stationarity ( $v_t = v_{t+1} \equiv v$ ), equation (3) gives

$$v = \frac{\rho R + (1 - \rho)\theta \ell}{\rho + (1 - \rho)\theta}. \quad (4)$$

Given the horizon mismatch assumption ( $\star$ ), the expression above is always less than the cost of investment  $c$ . Thus, if B borrows via a puttable loan, B cannot satisfy its borrowing constraint ( $\$$ ) and therefore cannot invest.

LEMMA 3. *If B borrows via a puttable loan, it cannot fund its investment.*

Note that the value of the puttable loan (equation (4)) is greater than the value of the standard loan (equation (2)). Thus, demandability (i.e. "puttability") is adding value and loosening B's borrowing constraint. This is because it offers  $C_0$  partial insurance against liquidity shocks: if  $C_0$  is hit by a liquidity shock, he cannot necessarily get the full repayment  $R$ , but he can still demand redemption and at least get  $\ell$ . This liquidity insurance may rationalize demandability in some circumstances. Indeed, it is reminiscent of the rationale for demandable debt in Calomiris and Kahn (1991), in that inefficient liquidation on the equilibrium path insures the creditor against bad outcomes.<sup>18</sup> In our setting, however, the horizon mismatch is so severe that early liquidation is relatively likely. Thus, it is too expensive for B to insure  $C_0$  by liquidating its investment whenever  $C_0$  needs liquidity. If B makes its debt tradeable, however, the secondary debt market provides  $C_0$  with insurance, even without liquidation. We turn to this next.

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<sup>18</sup>In Calomiris and Kahn (1991), "bad outcomes" are associated with moral hazard problems, rather than liquidity shocks.

### 3.4 Bond

Now we consider a bond, i.e. tradeable long-term debt. At Date  $t$ , the value  $v_t$  of the bond can be written recursively:

$$v_t = \rho R + (1 - \rho) \left( \theta \sigma_t p_t + (1 - \theta) v_{t+1} \right). \quad (5)$$

The terms are determined as follows. With probability  $\rho$ , B's investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B's investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. Since the bond is tradeable, but not demandable,  $H_t$  gets  $p_t$  if he finds a counterparty, which happens with probability  $\sigma_t$ , and nothing otherwise. With probability  $(1 - \rho)(1 - \theta)$ , B's investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains B's debt at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.

To solve for the value  $v_t$ , we must first give the secondary-market price of the bond  $p_t$ .

LEMMA 4. *The secondary-market price of the bond is  $p_t = v_t/2$ .*

The bond price splits the gains from trade fifty-fifty between  $H_t$  and  $C_t$ , given that  $H_t$  is hit by a liquidity shock. Since  $H_t$  has value zero in this case ( $H_t$  dies at the end of the period and the bond is not demandable), the gains from trade are just the value  $v_t$  of the bond to the new debtholder  $C_t$ .

By stationarity ( $v_t = v_{t+1} \equiv v$  and  $\sigma_t \equiv \sigma$ ), the preceding lemma implies  $p_t \equiv p \equiv v/2$ , so equation (5) gives

$$v = \frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)}. \quad (6)$$

Given the horizon mismatch assumption ( $\star$ ), the expression above is always less than the cost of investment  $c$ , even if the bond circulates (i.e.  $\sigma = 1$ ). Thus, if B borrows via a bond, B cannot satisfy its borrowing constraint ( $\$$ ) and therefore cannot invest.

LEMMA 5. *If B borrows via a bond, it cannot fund its investment.*

As in the case of the puttable loan above, the value of the bond (equation (6)) is greater than the value of the standard loan (equation (2)). Thus, tradeability is adding value, loosening B's borrowing constraint. This is because it offers  $C_0$  partial insurance against liquidity shocks: if  $C_0$  is hit by a liquidity shock, it cannot necessarily get the full repayment  $R$ , but it can still sell the bond in the secondary market and at least get  $p = v/2$ . This is the first step in showing that tradeability in the secondary market, or "market liquidity," helps loosen borrowing constraints in the primary market, or

improves “funding liquidity.” However, trading frictions in the OTC market depress the secondary-market bond price ( $p \ll v$ ). Thus, despite tradeability, liquidity shocks are costly for creditors and ultimately tradeability alone is not enough for B to get its investment up and running.

### 3.5 Banknote

Now we consider a banknote, i.e. tradeable, demandable debt. At Date  $t$ , the value  $v_t$  of the banknote can be written recursively:

$$v_t = \rho R + (1 - \rho) \left( \theta (\sigma_t p_t + (1 - \sigma_t) \ell) + (1 - \theta) v_{t+1} \right). \quad (7)$$

The terms are determined as follows. With probability  $\rho$ , B’s investment pays off and B repays  $R$ . With probability  $(1 - \rho)\theta$ , B’s investment does not payoff and the debtholder  $H_t$  is hit by a liquidity shock. Since the banknote is both tradeable and demandable,  $H_t$  gets  $p_t$  if he finds a counterparty, which happens with probability  $\sigma_t$ , and otherwise redeems on demand and gets  $\ell$ . With probability  $(1 - \rho)(1 - \theta)$ , B’s investment does not pay off and  $H_t$  is not hit by a liquidity shock.  $H_t$  retains the banknote at Date  $t + 1$ , which has value  $v_{t+1}$  at Date  $t$  since there is no discounting.

To solve for the value  $v_t$ , we must first give the secondary-market price of the banknote  $p_t$ .

LEMMA 6. *The secondary-market price of the banknote is  $p_t = (v_{t+1} + \ell)/2$ .*

The price of the banknote splits the gains from trade fifty-fifty between  $H_t$  and  $C_t$ , given that  $H_t$  is hit by a liquidity shock. Since  $H_t$  has value  $\ell$  ( $H_t$  redeems on demand and gets  $\ell$  if he does not trade with  $C_t$ ), the gains from trade are  $v_t - \ell$ , the value to the new debtholder  $C_t$  minus the value to the current debtholder  $H_t$ . The price that splits these gains is  $p_t = \ell + (v_t - \ell)/2 = (v_t + \ell)/2$ . Critically, the secondary-market price of the banknote is higher than the secondary-market price of the bond. This is because the option to redeem on demand improves  $H_t$ ’s bargaining position, since  $H_t$  gets a higher payoff if bargaining breaks down. Even if no debtholder ever demands redemption from B in the primary market, the option to do so can have important implications for the price of debt in the OTC secondary market.

This result suggests that the more costly it is to liquidate (the lower is  $\ell$ ), the lower is the secondary-market price  $p_t$ . This may cast light on the fact that nineteenth-century banknotes traded at a discount in markets far from the issuing bank, and that this discount was increasing in the distance to the issuer (Gorton (1996)): the farther you were from the issuer, the costlier it was for you to liquidate on demand and thus the weaker your bargaining position in the OTC market.



By stationarity ( $v_t = v_{t+1} \equiv v$  and  $\sigma_t \equiv \sigma$ ), the preceding lemma implies that  $p_t \equiv p \equiv (v + \ell)/2$ , so equation (7) gives

$$v = \frac{\rho R + (1 - \rho)\theta(1 - \sigma/2)\ell}{\rho + (1 - \rho)\theta(1 - \sigma/2)}. \quad (8)$$

Despite the horizon mismatch assumption ( $\star$ ), the expression above may be greater than the cost of investment  $c$ . Thus, if B borrows via a banknote, B may be able to satisfy his borrowing constraint ( $\$$ ) and therefore invest.

PROPOSITION 1. (DEMANDABILITY INCREASES DEBT CAPACITY.) *Suppose that*

$$\frac{1}{\theta} \frac{2(y - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}. \quad (9)$$

*If B borrows via a banknote and the banknote circulates ( $\sigma = 1$ ), then B can fund its investment.*

B can increase its debt capacity by borrowing via demandable debt, even if it is never redeemed on demand on the equilibrium path. This proposition thus suggests a new rationale for demandable debt: it props up the price in secondary markets and thereby loosens borrowing constraints in primary markets. Demandability increases the market liquidity of B's debt, which provides creditors with insurance against liquidity shocks in the future. This makes them more willing to provide B with funding liquidity today. Thus, market liquidity and funding liquidity are complements and, likewise, tradeability and demandability are complements.

### 3.6 Money Runs

Having established that B can borrow only with tradeable, demandable debt (banknotes), we now turn to secondary-market liquidity and the possibility that a debtholder demands early redemption. To do this, we assume that B has issued a banknote with promised repayment  $R$  and look at the equilibria of the subgames for  $t > 0$ . (We determine  $R$  in equilibrium in Proposition 3).

First, observe that B's banknote indeed circulates as long as  $\sigma_t = 1$  is a best response to the belief that  $C_{t'}$  plays  $\sigma_{t'} = 1$  for all  $t' > t$ . This is the case as long as  $C_t$  is willing to pay the entry cost  $k$  to gain the surplus  $v - p$  given  $\sigma = 1$ , or

$$k \leq v - p \Big|_{\sigma=1} = \frac{\rho(R - \ell)}{2\rho + (1 - \rho)\theta}, \quad (10)$$

having substituted in from Lemma 6 and equation (8).

But there may also be another equilibrium in which B's banknote does not circulate. B's banknote does not circulate as long as  $\sigma_t = 0$  is a best response to the belief that  $C_{t'}$  plays  $\sigma_{t'} = 0$  for all  $t' > t$ . This is the case as long as  $C_t$  is *not* willing to pay the entry cost  $k$  to gain the surplus  $v - p$  given  $\sigma = 0$ , or

$$k \geq v - p \Big|_{\sigma=0} = \frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta}, \quad (11)$$

again having substituted in from Lemma 6 and equation (8).

PROPOSITION 2. (MONEY RUNS.) *Suppose that B borrows via a banknote with promised repayment  $R$  and the entry cost  $k$  is such that*

$$\frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho(R - \ell)}{2\rho + (1 - \rho)\theta}. \quad (12)$$

*The  $t > 0$  subgame has both an equilibrium in which B's debt circulates ( $\sigma = 1$ ) and there is no early liquidation and an equilibrium in which B's debt does not circulate ( $\sigma = 0$ ) and there is early liquidation.*

Thus demandable debt has a dark side: if a counterparty  $C_t$  doubts future liquidity, i.e. he doubts that he will find a counterparty in the future, then  $C_t$  will not enter. As a result, the debtholder  $H_t$  indeed will not find a counterparty. There is a self-fulfilling dry-up of secondary-market liquidity. With demandable debt, this has severe real effects: unable to trade,  $H_t$  redeems its debt on demand, leading to costly liquidation of B's investment. In other words, a change in just the beliefs about future liquidity leads to the failure of B's debt as a medium of exchange in the secondary market—the failure of B's debt as money. As a result, there is sudden withdrawal of liquidity from B, i.e. a bank run, or a *money run*.

COROLLARY 1. *Suppose  $k$  satisfies condition (12). If  $C_t$ 's beliefs change from  $\sigma_{t'} = 1$  to  $\sigma_{t'} = 0$  for  $t' > t$ , the debtholder  $H_t$  “runs” on B, i.e.  $H_t$  unexpectedly demands redemption of his debt, forcing B to liquidate its investment.*

Demandability cuts both ways in the secondary market. It increases B's debt capacity by creating market liquidity, propping up the price of B's debt. But it also exposes B to money runs, since B must provide liquidity on demand if market liquidity dries up. Thus, financial fragility may be a necessary evil, resulting from the need to overcome funding constraints given a horizon mismatch between the investment and the creditors' liquidity needs.

### 3.7 Equilibrium Runs

We now turn to characterizing an equilibrium in which B borrows via a banknote and money runs arise on the equilibrium path. To do this, we expand the model slightly to introduce a “sunspot” coordination variable at each date,  $s_t \in \{0, 1\}$ . We will interpret  $s_t = 1$  as “normal times” and  $s_t = 0$  as a “confidence crisis,” since the sunspot does not affect economic fundamentals, but serves only as a way for agents to coordinate their beliefs. We assume that  $s_0 = 1$ , that  $\mathbb{P}[s_{t+1} = 0 | s_t = 1] =: \lambda$ , and that  $\mathbb{P}[s_{t+1} = 0 | s_t = 0] = 1$ , where we think about  $\lambda$  as a small number. In words: the economy starts in normal times and a permanent confidence crisis occurs randomly with small probability  $\lambda$ .

We now look for a Markov equilibrium, i.e. an equilibrium in which the sunspot (rather than the whole history) is a sufficient statistic for  $C_t$ 's action:

$$\sigma_t = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \quad (13)$$

Note that the baseline case of a stationary equilibrium is the special case of  $\lambda = 0$ . We can now write the banknote's value  $v^0$  when  $s_t = 0$  and  $v^1$  when  $s_t = 1$  (cf. the analogous equation for the stationary case in equation (7)):

$$v^0 = \rho R + (1 - \rho) \left( \theta \left( \sigma^0 p^0 + (1 - \sigma^0) \ell \right) + (1 - \theta) v^0 \right), \quad (14)$$

$$v^1 = \rho R + (1 - \rho) \left( \theta \left( \sigma^1 p^1 + (1 - \sigma^1) \ell \right) + (1 - \theta) \left( \lambda v^0 + (1 - \lambda) v^1 \right) \right). \quad (15)$$

The next proposition characterizes an equilibrium in which the “confidence crisis” induces a money run.

**PROPOSITION 3. (EQUILIBRIUM WITH SUNSPOT RUNS.)** *Suppose that the condition in equation (9) is satisfied. As long as  $\lambda$  is sufficiently small, there exists  $k$  such that B can fund its investment only with tradeable, demandable debt (a banknote), even though it admits a money run when  $s_t = 0$ . Specifically,  $C_t$  plays  $\sigma_t = s_t$ , and the value of the banknote when  $s_t = 0$  is*

$$v^0 = \frac{\rho R + (1 - \rho) \theta \ell}{\rho + (1 - \rho) \theta} \quad (16)$$

*the value of the banknote when  $s_t = 1$  is*

$$v^1 = \frac{\rho R + (1 - \rho) \left( \theta \ell / 2 + (1 - \theta / 2) \lambda v_0 \right)}{\rho + (1 - \rho) \left( \theta / 2 + (1 - \theta / 2) \lambda \right)}, \quad (17)$$

and the promised repayment is

$$R = y - \frac{(\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda)}{\rho(\rho + (1 - \rho)(\theta + (1 - \theta)\lambda))}(v^1 - c). \quad (18)$$

We have written the system recursively, expressing  $v^1$  as a function of  $v^0$  and  $R$  as a function of  $v^1$ , for simplicity. We give a closed-form expression for  $R$  in terms of primitives in Appendix 3.

### 3.8 Circulating Bonds and Liquidity Dry-ups without Runs

So far, we have maintained the horizon mismatch assumption ( $\star$ ), which implied that only the banknote was feasible. We relax this assumption now, so that bonds are feasible too. We find that bonds have two advantages over banknotes: (i) they are more liquid in the secondary market, which can help with financing, and (ii) they are not subject to runs, which can prevent inefficient liquidation.

PROPOSITION 4. (BOND FINANCE.) *Suppose that*

$$\frac{1}{\theta} \frac{y - c}{(c - \ell)(1 - \rho)} \leq \frac{1}{\rho} \leq \frac{1}{\theta} \frac{2(y - c)}{c(1 - \rho)}, \quad (19)$$

and

$$\frac{\rho(y - \ell)}{2\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho y}{2\rho + (1 - \rho)\theta}. \quad (20)$$

*B can fund its investment only with tradeable, long-term debt (a bond).*

Intuitively, this result says that if there is a horizon mismatch which is not too severe (condition (19)), then B can borrow via a bond, but cannot borrow via a non-circulating instrument, such as a puttable loan. And, further, if the entry cost  $k$  is large enough (condition (20)), then B cannot borrow via a banknote either. The reason is that demandability increases secondary-market prices, which can discourage entry: whereas a counterparty may be willing to pay  $k$  to enter the bond market, since he anticipates buying at a discount, he may be reluctant to pay  $k$  to enter the banknote market, since he anticipates paying a high price. Thus, the banknote may not circulate—and a non-circulating banknote is effectively just a puttable loan.

Even though a counterparty is relatively willing to pay  $k$  to enter the bond market, liquidity dry-ups can still occur.

COROLLARY 2. *Suppose that B borrows via a bond with promised repayment  $R$  and that*

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho R}{2\rho + (1 - \rho)\theta}. \quad (21)$$

The  $t > 0$  subgame has both an equilibrium in which  $B$ 's debt circulates ( $\sigma = 1$ ) and an equilibrium in which  $B$ 's debt does not circulate ( $\sigma = 0$ ). However, there is not early liquidation in either equilibrium;  $B$ 's investment always pays off  $y$ .

Like trade in banknotes, trade in bonds is self-fulfilling. A self-fulfilling liquidity dry-up can arise whenever a counterparty enters if and only if he believes that other counterparties will enter in the future (cf. equations (10) and (11) in Subsection 3.6). However, since the bond is not demandable, liquidity dry-ups in the secondary market do not lead to runs or inefficient liquidation of  $B$ 's investment.

The analysis above captures the two advantages that bonds have over banknotes: (i) dry-ups in bond liquidity are “less likely” than dry-ups in banknote liquidity, since counterparties enter the bond market for larger  $k$ , and (ii) they are less costly, since they do not lead to  $B$ 's liquidation. Thus, we expect to see demandable debt mainly when the horizon mismatch is so severe that borrowing via bonds is impossible. We think this casts light on why bank debt is more likely to be demandable than corporate debt: the horizon mismatch assumption ( $\star$ ) is more likely to hold for banks than for other firms.

### 3.9 Asset Choice

What if  $B$  can choose the type of its investment before borrowing from  $C_0$ ? Can frictions in the secondary market distort its choice? Yes, it is distorted toward high-liquidation-value investments, which can have higher debt capacity even when they have lower NPV. To see this, suppose that  $B$  can choose between an investment with payoff  $y$  and liquidation value  $\ell$  and another investment that is otherwise identical but has payoff  $y'$  and liquidation value  $\ell'$ .

PROPOSITION 5. (EXCESSIVE LIQUIDITY.) *Suppose that  $y > y'$  but*

$$\ell < \ell' - \frac{2\rho}{(1-\rho)\theta}(y - y'). \quad (22)$$

*There exists an investment cost  $c$  such that in any equilibrium in which investment occurs,  $B$  chooses the low-NPV, high-liquidation-value investment  $(y', \ell')$ .*

### 3.10 Intermediation

In the analysis so far, the borrower  $B$  resembles a bank only because it transforms maturity, financing a long-term investment with demandable debt. Now we go a step further, and argue that our model naturally gives rise to intermediated finance, in which  $B$  is not only a borrower but also a lender. Specifically, suppose that  $B$  has only assets

with liquidation value  $\ell$  and another player  $B'$  has the investment which pays off  $y$  with intensity  $\rho$ . Assume, however, that this investment has zero liquidation value, so  $B'$  cannot issue demandable debt. As a result,  $B'$  may be unable to get finance from  $C_0$ . But there is room for  $B$  to step in as an intermediary to “create liquid paper.” I.e.  $B$  lends to  $B'$  via a loan and borrows from  $C_0$  via a banknote backed by both  $B'$ 's debt and its own liquid assets  $\ell$ . Even though these assets create no real value, the option to redeem them on demand puts debtholders in a stronger bargaining position in the secondary market, making  $C_0$  willing to lend. This generates intermediation rent for  $B$ .<sup>19</sup>

PROPOSITION 6. (INTERMEDIATION.) *Suppose that*

$$\frac{1}{\theta} \frac{2(y - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho} \quad (23)$$

*as in Proposition 1 (and the horizon mismatch assumption  $(\star)$  holds). Direct finance in which  $B'$  borrows from  $C_0$  via any instrument is infeasible. However, intermediated finance in which  $B'$  borrows from  $B$  via a loan and  $B$  borrows from  $C_0$  via a banknote is feasible.*

### 3.11 Partial Rollover

We now turn to a version of the model with many debtholders and many counterparties at each date. In this set-up, some debtholders demand redemption at each date, so not every withdrawal is a run. But a run can still occur following a dry-up of secondary market liquidity, even though liquidity shocks are independent across debtholders. This affirms that our results are driven just by coordination, not by aggregate liquidity risk. Further, we find that runs can occur even if  $B$  can risklessly roll over its debt at each date, which distinguishes our run risk from rollover risk.<sup>20</sup>

Here we suppose that  $B$  has issued banknotes with face value  $R$  and early redemption value  $\ell$ . To focus on the secondary market circulation and rollover, we assume

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<sup>19</sup>The idea that banks emerge because they create private circulating debt also appears in Donaldson, Piacentino, and Thakor (2017), Gu, Mattesini, Monnet, and Wright (2013), and, in particular, Kiyotaki and Moore (2001a), who say

the trick would be to make a profit by buying blue paper [i.e. illiquid debt] and selling red [liquid debt]! Imagine...offering to lend to private people whose IOUs are illiquid. Since their paper is illiquid, blue, they have to pay you a relatively high rate of interest. Meantime, you raise funds by taking in deposits....You are making a profit, merely by sitting there! Do you know what you are? You are a bank. You effectively transform blue paper into red (page 23).

<sup>20</sup>We use “run risk” to mean the risk of an unexpectedly large number of withdrawals. In contrast, we use “rollover risk” to mean the risk that  $B$  attempts to raise new debt and fails. Below, we assume  $B$  can roll over costlessly—there is no rollover risk—but  $B$  cannot go back to the market to meet a large number of withdrawals without some delay—there is run risk.

that the banknotes are held by a unit continuum of debtholders; we do not model funding/investment. To model rollover, we assume that B matches with new creditors at the beginning of each date, raising  $\ell$  from each of them via new banknotes with face value  $R$  and redemption value  $\ell$ . Note that the amount raised and the redemption value coincide; this keeps the model stationary. Note, further, that B must decide how much to raise at the beginning of the date; this makes runs possible.

In the secondary market, there is a “large” continuum of counterparties at each date who can enter to trade with the debtholders at cost  $k$ . These creditors are matched with debtholders via a homogenous matching technology, as in the search-and-matching literature.  $\sigma_t$  denotes the probability that a debtholder is matched with a counterparty, as in the baseline model, except here it depends on the number of counterparties that enter. By the law of large numbers, a fraction  $\theta$  of debtholders are shocked at each date, of which  $\sigma_t$  are matched with counterparties. These debtholders trade in the secondary market. The remaining  $\theta(1 - \sigma_t)$  debtholders redeem at the bank for  $\ell$ . We assume that B’s rollover strategy is determined so as to meet these redemptions. I.e. it raises exactly  $\theta(1 - \sigma_t)\ell$  at the beginning of the date. This ensures that the mass of debtholders is constant at one.

The next result says that this set-up has multiple steady states. Indeed, there is a “good” equilibrium, in which many counterparties enter and few debtholders are left unmatched. In this equilibrium, there are relatively few withdrawals at each date, so B chooses its rollover strategy to raise a relatively small amount of liquidity. But there is also a “bad” equilibrium, in which few counterparties enter and many debtholders are left unmatched. In this equilibrium, there are more withdrawals at each date, so B has to choose a rollover strategy to raise more liquidity. Thus, a change in beliefs can lead to a money run: if counterparties believe that few counterparties will enter in the future, then few counterparties will enter today, leading to an unexpectedly high number of withdrawals—a money run forcing liquidation.

PROPOSITION 7. (MONEY RUNS WITH PARTIAL ROLLOVER.) *Let the matching technology be given by  $\sigma = \mu\sqrt{q}$ , where  $q$  is the number of counterparties that enter and  $\mu$  is sufficiently small. Suppose that B borrows via banknotes from a continuum of creditors. The  $t > 0$  subgame has two stationary equilibria, one in which  $\sigma$  is high—banknotes are liquid—and one in which  $\sigma$  is low—banknotes are illiquid and there are many withdrawals, i.e. there is a money run.*

Here, money runs can occur even with no aggregate risk, no rollover risk, and no sequential-service constraint. This affirms that money runs result only from intertemporal coordination in the secondary market, and distinguishes our model of bank fragility from the complementary models of rollover risk (e.g. He and Xiong (2012), Acharya, Gale, and Yorulmazer (2011)) and of bank runs (Diamond and Dybvig (1983)).

## 4 Conclusion

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One important function of banks is to create money, i.e. to issue debt that circulates in OTC markets. By focusing on this function of banks, we found a new type of bank runs—“money runs”—and a new rationale for demandable debt, both of which are the result of how bank debt circulates—or fails to circulate—in secondary markets. Money runs occur because secondary-market liquidity is fragile and self-fulfilling. Banks issue demandable debt because its secondary-market price is high. In contrast to the literature, our results suggest that financial fragility may be a necessary evil and, further, that regulating markets may help bank stability more than regulating banks themselves.



## A Proofs

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### A.1 Proof of Lemma 1

The result follows immediately from  $C_0$ 's participation constraint: B and  $C_0$  find an agreement point if and only if bargaining gives both more than their disagreement utilities. Since  $C_0$  receives  $v_0$  from bargaining, this must exceed its cost  $c$ .  $\square$

### A.2 Proof of Lemma 2

By Lemma 1, B can invest only if  $v_0 = v \geq c$  or, by equation (2),

$$\frac{\rho R}{\rho + (1 - \rho)\theta} \geq c. \quad (24)$$

This says that

$$\frac{1}{\theta} \frac{R - c}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (25)$$

which violates the assumption  $(\star)$  since  $R \leq y$ .  $\square$

### A.3 Proof of Lemma 3

By Lemma 1, B can invest only if  $v_0 = v \geq c$  or, by equation (4),

$$\frac{\rho R + (1 - \rho)\theta \ell}{\rho + (1 - \rho)\theta} \geq c. \quad (26)$$

Since  $\ell < c/2$ , a necessary condition for this is that

$$\frac{\rho R + (1 - \rho)\theta c/2}{\rho + (1 - \rho)\theta} \geq c. \quad (27)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (28)$$

which violates the assumption  $(\star)$  since  $R \leq y$ .  $\square$

### A.4 Proof of Lemma 4

When  $C_t$  and  $H_t$  are matched  $H_t$  has been hit by a liquidity shock. Thus,  $C_t$ 's value of the bond is  $v_t$  and  $H_t$ 's value of the bond is zero (since  $H_t$  consumes only at Date  $t$  and the bond is not demandable). The total surplus is thus  $v_t$ , which  $C_t$  and  $H_t$  split fifty-fifty in accordance with the Nash bargaining solution. Thus the price is  $p_t = v_t/2$ .

## A.5 Proof of Lemma 5

By Lemma 1, B can invest only if  $v_0 = v \geq c$  or, by equation (6),

$$\frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq c. \quad (29)$$

This is increasing in  $\sigma$ , so a necessary condition is that the above holds for  $\sigma = 1$ , or

$$\frac{\rho R}{\rho + (1 - \rho)\theta/2} \geq c. \quad (30)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)} \geq \frac{1}{\rho} \quad (31)$$

which violates the assumption  $(\star)$  since  $R \leq y$ .  $\square$

## A.6 Proof of Lemma 6

When  $C_t$  and  $H_t$  are matched  $H_t$  has been hit by a liquidity shock. Thus,  $C_t$ 's value of the banknote is  $v_t$  and  $H_t$ 's value of the banknote is  $\ell$  (since  $H_t$  consumes only at Date  $t$ , it redeems on demand if it does not trade). The gains from trade are thus  $v_t - \ell$ , which  $C_t$  and  $H_t$  split fifty-fifty in accordance with the Nash bargaining solution, i.e.  $p_t$  is such that

$$H_t \text{ gets } \frac{1}{2}(v_t - \ell) + \ell = p_t, \quad (32)$$

$$C_t \text{ gets } \frac{1}{2}(v_t - \ell) = v_t - p_t, \quad (33)$$

or  $p_t = (v_t + \ell)/2$ .

## A.7 Proof of Proposition 1

By Lemma 1, B can invest if  $v_0 = v \geq c$  or, by equation (8) with  $\sigma = 1$ ,

$$\frac{2\rho R + (1 - \rho)\theta\ell}{2\rho + (1 - \rho)\theta} \geq c. \quad (34)$$

This says that

$$\frac{1}{\theta} \frac{2(R - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}. \quad (35)$$

This is feasible for some  $R \leq y$ , i.e. as long as

$$\frac{1}{\theta} \frac{2(y - c)}{(c - \ell)(1 - \rho)} \geq \frac{1}{\rho}, \quad (36)$$

which is the condition in the proposition (note this is mutually compatible with the horizon mismatch assumption  $(\star)$ ).  $\square$

## A.8 Proof of Proposition 2

The argument is in the text.

## A.9 Proof of Corollary 1

The result follows immediately from Proposition 2.

## A.10 Proof of Proposition 3

We first solve for the values  $v^0$  and  $v^1$  given the strategies  $\sigma^0 = 0$  and  $\sigma^1 = 1$ . We then show that these strategies are indeed best responses (for some  $k$ ). Finally, we compute the repayment  $R$  in accordance with the Nash bargaining solution.

**Values.** From equation (14) with  $\sigma^0 = 0$ , we have immediately that

$$v^0 = \frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \quad (37)$$

(this is just the value of the puttable loan in equation (4)). From Lemma 6 (the logic of which is not affected by the presence of sunspots), we have the price

$$p^1 = \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2}. \quad (38)$$

Thus, equation (15) with  $\sigma^1 = 1$  reads

$$v^1 = \rho R + (1 - \rho) \left( \theta \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2} + (1 - \theta) (\lambda v^0 + (1 - \lambda)v^1) \right), \quad (39)$$

so

$$v^1 = \frac{\rho R + (1 - \rho) (\theta\ell/2 + (1 - \theta/2)\lambda v_0)}{\rho + (1 - \rho) (\theta/2 + (1 - \theta/2)\lambda)} \quad (40)$$

**Best responses.**  $\sigma^1 = 1$  and  $\sigma^0 = 0$  are best response if

$$v^0 - p^0 \leq k \leq v^1 - p^1 \quad (41)$$

or

$$v^0 - \frac{v^0 + \ell}{2} \leq k \leq v^1 - \frac{\lambda v^0 + (1 - \lambda)v^1 + \ell}{2}. \quad (42)$$

This is satisfied for some  $k$  as long as  $v^1 \geq v^0$ , which is the case as long as  $R \geq \ell$ , which must be the case.

**Repayment.** The repayment  $R$  is determined by Nash bargaining between B and  $C_0$ . Now denote B's value in state  $s$  by  $u^s$ . Since the state is  $s = 1$  at Date 0, B and  $C_0$  agree on a repayment  $R$  such that B's value is  $u^1$  and  $C_0$ 's value is  $v^1$ . The gains from trade are thus  $u^1 + v^1 - c$ .  $R$  is thus such that

$$\text{B gets } \frac{1}{2}(u^1 + v^1 - c) = u^1, \quad (43)$$

$$\text{C}_0 \text{ gets } \frac{1}{2}(u^1 + v^1 - c) + c = v^1, \quad (44)$$

so  $u^1 = v^1 - c$ .

From the expressions above we can compute  $R$  in terms of primitives. First we compute B's value  $u^0$  in state 0. In this case, the banknote does not circulate and the banknote is effectively like the puttable loan in Subsection 3.3. Whenever the creditor is hit by a liquidity shock, he redeems on demand and B liquidates and gets zero. B's value is thus

$$u^0 = \rho(y - R) + (1 - \rho)(1 - \theta)u^0 \quad (45)$$

or

$$u^0 = \frac{\rho(y - R)}{\rho + (1 - \rho)\theta}. \quad (46)$$

Now, B's value in state 1 is given by

$$u^1 = \rho(y - R) + (1 - \rho)\left((1 - \lambda)u^1 + \lambda(1 - \theta)u^0\right) \quad (47)$$

or, substituting in for  $u^0$  from equation (46) above,

$$u^1 = \frac{\rho(y - R)}{\rho + (1 - \rho)\lambda} \left(1 + \frac{(1 - \rho)\lambda(1 - \theta)}{\rho + (1 - \rho)\theta}\right). \quad (48)$$

Substituting for this into the bargaining outcome  $u^1 = v^1 - c$  gives the expression for the repayment  $R$  in terms of  $v_1$  as stated in the proposition.

Now we can also substitute for  $v^1$  from equation (40) and  $v_0$  from equation (37) above to get

$$R = \frac{Ay + Bc - C\ell}{D} \quad (49)$$

where the constants  $A$ ,  $B$ ,  $C$ , and  $D$  are defined as follows:

$$\begin{aligned} A &= \left(2(\rho + (1 - \rho)\lambda + (1 - \rho)(1 - \lambda)\theta)\right) \left((\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda) - (1 - \rho)\theta\lambda\right), \\ B &= \left(2(\rho + (1 - \rho)\lambda + (1 - \rho)(1 - \lambda)\theta)\right) (\rho + (1 - \rho)\theta)(\rho + (1 - \rho)\lambda), \\ C &= (\rho + (1 - \rho)\lambda)(1 - \rho) \left(\rho + (1 - \rho)(2\lambda + (1 - \lambda)\theta)\right), \\ D &= \rho \left(\theta(5 + 4\lambda)(1 - \rho)(\rho + (1 - \rho)\lambda) + \theta^2(1 - \lambda)^2(1 - \rho)^2 + 4(\rho + (1 - \rho)\lambda)^2\right). \end{aligned}$$

### A.11 Proof of Proposition 4

We first show that for  $k$  satisfying condition (20), the bond can circulate but the banknote cannot. We then show that if the bond circulates, B can fund its investment with a bond. However, if the banknote does not circulate, B cannot fund its investment with a banknote. This also implies that B cannot fund its investment with a puttable loan or a loan: a puttable loan is identical to a non-circulating banknote, which is infeasible, and a puttable loan always has higher debt capacity than a loan (cf. equations (2) and (4)).

**Bond can circulate.** The bond circulates whenever  $C_t$ 's value  $v$  less the price  $p$  exceeds his cost of entry  $k$ , or, given the value in equation (6) and the price in Lemma 4,

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq k. \quad (50)$$

For  $R = y$  and  $\sigma = 1$  this reads

$$\frac{\rho y}{2\rho + (1 - \rho)\theta} \geq k, \quad (51)$$

which is satisfied by assumption (condition (20)). Thus, the bond can circulate as long as  $R$  is sufficiently large.

**Banknote cannot circulate.** The banknote circulates whenever  $C_t$ 's value  $v$  less the price  $p$  exceeds his cost of entry  $k$ , or, given the value in equation (8) and the price in Lemma 6,

$$\frac{1}{2} \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta(1 - \sigma/2)} \geq k. \quad (52)$$

The left-hand side is maximized when  $R = y$  and  $\sigma = 1$ , for which the inequality reads

$$\frac{\rho(y - \ell)}{2\rho + (1 - \rho)\theta} \geq k, \quad (53)$$

which is never satisfied by assumption (condition (20)). Thus, the banknote cannot circulate.

**Bond financing feasible.** Given the bond circulates ( $\sigma = 1$ ), the borrowing

constraint (\$) is satisfied whenever

$$\frac{\rho R}{\rho + (1 - \rho)\theta/2} \geq c, \quad (54)$$

given the value of the bond from equation (6). This can be re-written as

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{2(R - c)}{c(1 - \rho)}. \quad (55)$$

This is satisfied by assumption (condition (19)) as long as  $R$  is sufficiently large.

**Banknote financing infeasible.** Given the banknote does not circulate ( $\sigma = 0$ ), the borrowing constraint (\$) is satisfied whenever

$$\frac{\rho R + (1 - \rho)\theta\ell}{\rho + (1 - \rho)\theta} \geq c, \quad (56)$$

given the value of the banknote from equation (8). This can be re-written as

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{R - c}{(c - \ell)(1 - \rho)}. \quad (57)$$

The right-hand side is maximized when  $R = y$ , for which the inequality reads

$$\frac{1}{\rho} \leq \frac{1}{\theta} \frac{y - c}{(c - \ell)(1 - \rho)}. \quad (58)$$

This is never satisfied by assumption (condition (19)). □

## A.12 Proof of Corollary 2

The argument here is analogous to the argument for the banknote in Subsection 3.6: there are multiple equilibria whenever (i) a counterparty entering is a best response other counterparties entering and (ii) a counterparty not entering is a best response to other counterparties not entering, or

$$v - p \Big|_{\sigma=0} \leq k \leq v - p \Big|_{\sigma=1}. \quad (59)$$

Substituting in for the value of the bond  $v$  from equation (6) and the price of the bond  $p$  from Lemma 4, this reads

$$\frac{1}{2} \frac{\rho R}{\rho + (1 - \rho)\theta} \leq k \leq \frac{\rho R}{2\rho + (1 - \rho)\theta}, \quad (60)$$

as stated in the corollary. □

### A.13 Proof of Proposition 5

As in the proof of Proposition 1, we have that, by Lemma 1, B can invest in  $(y', \ell')$  but not in  $(y, \ell)$  if and only if the value of the banknote with repayment  $y$  and liquidation value  $\ell$  is less than the investment cost  $c$ , which is in turn less than the value of the banknote with repayment  $y'$  and liquidation value  $\ell'$ . By equation (8) with  $\sigma = 1$ , this says that

$$\frac{\rho y + (1 - \rho)\theta\ell/2}{\rho + (1 - \rho)\theta/2} < c < \frac{\rho y' + (1 - \rho)\theta\ell'/2}{\rho + (1 - \rho)\theta/2} \quad (61)$$

There exists  $c$  satisfying the above inequalities whenever the left-most term is less than the right-most term. This reduces to the condition in the proposition (equation (22)).  $\square$

### A.14 Proof of Proposition 6

Recall first that B's investment has zero liquidation value and observe that a banknote with zero liquidation value is effectively a bond. Thus, the fact that direct finance is infeasible follows from Lemma 5, which says that bond finance is infeasible given the horizon mismatch assumption  $(\star)$ .

Now observe that if B makes a loan to B' with face value  $R'$ , then B can borrow from  $C_0$  via a banknote with liquidation value  $\ell$  and any face value  $R \leq R'$ . Given the assumption in the proposition (equation (23)), this is feasible for  $R'$  sufficiently close to  $y$  by Proposition 1. Since limited liability is the only restriction on repayments, any repayments  $R \leq R' \leq y$  are feasible. Thus this intermediation arrangement is feasible.  $\square$

### A.15 Proof of Proposition 7

Observe first that the value of the banknote is given by the same expression as in the baseline model (equation (8)), since  $\sigma$  still represents the probability that a debtholder finds a counterparty. Now,  $\sigma$  is determined by counterparties' entry condition. Recall that the matching function is homogenous, so each counterparty is matched with a debtholder with probability  $\sigma/q$ . Counterparties' entry condition is thus

$$\frac{\sigma}{q}(v - p) \geq k \quad (62)$$

where  $q$  represents the steady-state mass of counterparties entering at each date. Since each counterparty is small, the inequality above will bind. So, substituting in for  $v$  and

$p = (v + \ell)/2$ , we have

$$\frac{\sigma}{2q} \left( \frac{\rho(R - \ell)}{\rho + (1 - \rho)\theta(1 - \sigma(q)/2)} \right) = k. \quad (63)$$

With  $\sigma = \mu\sqrt{q}$ , this can be re-written as

$$\mu k(1 - \rho)q - 2k(\rho + (1 - \rho)\theta)\sqrt{q} + \mu\rho(R - \ell) = 0. \quad (64)$$

This is a quadratic equation in  $\sqrt{q}$ . It has the two solutions, i.e. there are two steady states,

$$q_{\pm} = \frac{k(\rho + (1 - \rho)\theta) \pm \sqrt{k^2(\rho + (1 - \rho)\theta)^2 - \mu^2 k \rho(1 - \rho)(R - \ell)}}{\mu k(1 - \rho)}, \quad (65)$$

as long as the discriminant above and the matching probabilities  $\sigma$  and  $\sigma/q$  are well defined. These conditions are satisfied as long as  $\mu$  is sufficiently small, as required in the statement of the proposition.  $\square$



## B Table of Notations

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Players and Indices	
$t$	time index
$B$	borrower or “bank”
$C_t$	(potential) creditor/counterparty at Date $t$
$H_t$	debtholder at Date $t$
Technologies and Preferences	
$y$	payoff of B’s investment
$c$	cost of B’s investment
$\ell$	liquidation value of B’s investment
$\rho$	probability B’s investment pays off each date
$\theta$	probability with which creditor is hit by liquidity shock at each date
Prices, Values, and Strategies	
$R$	B’s promised repayment (face value of debt)
$v_t$	value of B’s debt to a creditor at Date $t$
$p_t$	secondary-market price of B’s debt at Date $t$
$\sigma_t$	mixed strategy of counterparty $C_t$
Other Variables	
$s_t$	sunspot at Date $t$ (Subsection 3.7)
$\lambda$	$\mathbb{P}[s_{t+1} = 0   s_t = 1]$ , “confidence crisis” probability (Subsection 3.7)

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