MONEY RUNS*

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Abstract

We present a banking model in which bank debt circulates as "money" in decentralized secondary markets, like banknotes did in the nineteenth century and repos do today. We find that bank debt is susceptible to runs because secondary-market liquidity is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a "money run." Even though demandable debt exposes banks to costly runs, banks still want to issue it. Indeed, to facilitate creating demandable money, banks in our model pool investments and transform maturity/liquidity—they endogenously do something that resembles real-world banking. Thus, the model suggests a raison d'être for banking based on money creation alone.

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In the use of money, every one is a trader.

David Ricardo (1876)

1 Introduction

Bank debt was a major form of money in the nineteenth-century United States. To get beer from the barman, you would exchange private banknotes over the counter (OTC). Banknotes were redeemable on demand and sudden redemptions—bank runs—were common.[1] If you held bank debt, you could get liquidity either by trading OTC or, alternatively, by demanding redemption from the issuing bank. But demanding redemption comes with the risk of a run. Why would you run on a bank rather than trade its debt in the market? In other words, why is bank debt susceptible to costly runs, even though it is tradeable? Moreover, why do banks choose to borrow via demandable debt, even though it exposes them to costly runs?

To give a new perspective on these questions, we focus on how banks create money by issuing liabilities that circulate in OTC markets, like banknotes did in the nineteenth century and much bank debt does today (see below). In the model, bank debt is susceptible to runs because liquidity in the OTC market is subject to sudden, self-fulfilling dry-ups. When debt fails to circulate it is redeemed on demand in a bank run, or "money run." Such runs were common in the nineteenth-century US, when depositors ran on banks after "the bank note that passed freely yesterday was rejected this morning."[2] Banks still choose to issue

[1] Gorton (2012b) argues that it remains a theoretical challenge to understand how these runs arise and how they affect the design of bank liabilities that circulate as money. He suggests that, because banknotes and repos are backed by collateral, there is no "common pool problem" inducing depositors to race to withdraw first in the event of a crisis:

> In the U.S. under state free banking laws banks were required to back their notes with state bonds. In the case of a bank failure—an inability to honor requests for cash from noteholders—the state bonds would be sold (by the state government) and the note holders paid off pro rata. Note holders were paid off pro rata, so there was no common pool problem. Yet, there was a run on banks (banknotes and deposits) during the Panic of 1857 (p. 15).

And he goes on to say:

> Generating such [a run] event in a model seems harder when...the form of money [is such that] each "depositor" receives a bond as collateral. There is no common pool of assets on which bank debt holders have a claim. So, strategic considerations about coordinating with other agents do not arise. This is a challenge for theory and raises issues concerning notions of liquidity and collateral, and generally of the design of trading securities—private money (p. 2).

We generate such runs on bank debt in a model in which banks optimally design securities that circulate in secondary markets. These runs occur because strategic considerations about coordinating with other agents do arise in the secondary market. (Other papers, such as Martin, Skeie, and von Thadden (2014a, 2014b) and Kuong (2015) study other mechanisms by which runs on backed debt can occur.)

[2] Treasury Secretary Howell Cobb (1858), quoted in Gorton (2012a), p. 36. Cobb goes on to suggest that

demandable debt even though it exposes them to costly runs. In our model, the reason is that it increases their debt capacity, since the option to redeem on demand increases the price it trades at—unlike in Jacklin (1987), demandability and tradeability are complements. To create this circulating demandable debt, banks pool investments and transform maturity/liquidity (even absent the benefits of diversification emphasized in the literature). I.e., just to create money, they do something that looks like real-world banking. But, to do it effectively, they cannot hold enough liquidity to meet all redemptions at once. Hence, bank fragility is a necessary evil. Overall, our model reveals a new type of run, a new rationale for demandable debt, and a new raison d'être for banking, all of which are based on how bank liabilities circulate as money in the secondary market.

Model preview. A borrower B has an investment opportunity and needs to borrow from a creditor C_0 to fund it. The model is based on two key assumptions. First, there is a horizon mismatch, similar to that in Diamond and Dybvig (1983): C_0 may be hit by a liquidity shock before B's investment pays off. Second, B's debt is traded in an OTC market, similar to those in Trejos and Wright (1995) and Duffie, Gârleanu, and Pedersen (2005): if C_0 is hit by a liquidity shock before B's investment pays off, C_0 can match with a counterparty C_1 and bargain bilaterally to trade B's debt. Likewise, C_1 may be hit by a liquidity shock before B's investment pays off, in which case he can match with a counterparty C_2 and bargain bilaterally to trade B's debt, and so on. If B's debt is demandable, then a creditor may redeem it before the investment pays off, forcing B to liquidate inefficiently to pay the redemption value.

Results preview. Our first main result is that B's debt capacity is highest if it issues tradeable, demandable debt, which we refer to as a "banknote." In particular, as long as the horizon mismatch is sufficiently severe, B cannot fund its investment with non-tradeable debt (e.g., a bank loan), even if it is demandable, or with non-demandable debt (e.g., a bond), even if it is tradeable. To see why, consider C_0's decision whether or not to lend to B. C_0 knows that he may be hit by a liquidity shock before B's investment pays off, in which case C_0 liquidates B's debt, either by redeeming on demand or by trading in the OTC market. If B's debt is not tradeable (but is demandable), then C_0 must redeem on demand, forcing B into inefficient liquidation and recovering less than his initial investment. If the horizon mismatch is severe, then this loss from early redemption is so likely that C_0 is unwilling to lend in the first place. In contrast, if B's debt is tradeable (but is not demandable), then C_0 can avoid early redemption by trading with C_1 in the OTC market. However, C_0's liquidity

the failure of banks can cause the failure of their debt to circulate as money. We emphasize that the chain of causation can run in the other direction, in line with Gorton's (2012a) interpretation of bank panics in the National Banking Era, when "the fear of a sudden appearance of a discount on checks [i.e. on bank money] led to bank runs" (p. 21).

shock puts him in a weak bargaining position with C_1: C_0 has a low outside option because he has no way to get liquidity if trade fails. As a result, he sells B's debt at a discounted price, recovering less than his initial investment. If the horizon mismatch is severe, then this loss from selling at a discount is so likely that C_0 is unwilling to lend in the first place. But if B's debt is demandable as well as tradeable, then debt does not trade at such a high discount in the secondary market. This is because demandability improves C_0's bargaining position with C_1. It increases his outside option, since he can redeem on demand when trade fails. As a result, C_0 can trade B's debt at a high price following a liquidity shock. Thus, C_0 is insured against liquidity shocks, making him willing to fund B's investment. This result contrasts with existing models of demandable debt, in which, roughly, you do not need the option to redeem debt on demand if you can just trade it in the secondary market (e.g., Jacklin (1987)). Here, in contrast, you do: just the option to redeem on demand props up the resale price of debt in the secondary market, even if the debt is never actually redeemed on demand in any state of the world.

Our second main result is that banknotes are susceptible to a new kind of bank run, which results directly from the dry-up of secondary-market liquidity. Specifically, a sudden (but rational) change in beliefs can cause secondary-market trading to stop, leading the creditor to redeem on demand and forcing B to liquidate inefficiently to pay the redemption value. The belief change may be precipitated by a shock to fundamentals, in which case the run amplifies a downturn, or by a "sunspot" unrelated to fundamentals, in which case the run constitutes a panic in itself. Either way, the run occurs even though B has only a single creditor—there is no static coordination problem in which multiple creditors race to withdraw as in Diamond and Dybvig (1983); rather, there is a dynamic coordination problem in the secondary market in which a counterparty does not accept B's debt today because he is worried that his future counterparty will not accept B's debt tomorrow. Due to this self-fulfilling liquidity dry-up, B's creditor is suddenly unable to trade when he is hit by a liquidity shock and, thus, he must demand redemption from B. We refer to this run as a "money run" because it is the result of the failure of B's debt to function as a liquid money in the secondary market.

These first two results imply that demandability cuts both ways. It is both the thing that allows B to fund itself and the thing that exposes B to runs. Thus, B faces a tradeoff when it sets the redemption value. A high redemption value helps C_0 to extract a high price from C_1, which makes C_0 more willing to lend. But this high price also increases the likelihood that C_1 chooses not to trade, which makes C_0 more likely to run. Despite these countervailing forces, B may still set the maximum possible redemption value (equal to the total liquidation value of its investment). Why? Becuase B gets the full benefit of cheaper borrowing, but does not bear the full cost of money runs. Hence, although financial fragility may be necessary—B must make its debt demandable to invest efficiently—it can also be excessive—B makes the redemption value too high, exposing itself to more runs than necessary.

For our third main result, we suppose that the horizon mismatch is so severe that B cannot fund its investment, even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is? To address this question, we consider N parallel versions of the model—N parallel borrowers borrow from N parallel creditors who trade in N parallel OTC markets. We assume that both borrowers' investments and creditors' liquidity shocks are perfectly correlated, so there is no possibility of diversification. But we find that the borrowers can still benefit from pooling their investments. They can issue N banknotes, each backed by the entire pool, and thereby "reuse" the investments' liquidation values—each banknote's redemption value is no longer bounded by the liquidation value of a single investment, but by the liquidation value of the pool. The borrowers cannot honor all the banknotes at once in equilibrium, but they do not have to: the debtholders' option to redeem them off equilibrium is enough to increase the banknotes' secondary market price and hence the borrowers' debt capacity. Even absent diversification, these borrowers look like a bank. In fact, based only on their need to create demandable debt, or "money," they endogenously transform liquidity and maturity, pool assets, and have dispersed creditors. And, like a bank, they are fragile. Given the high redemption values, prices are high and counterparties are reluctant to trade. Thus the "bank" is vulnerable to money runs, which can trigger liquidation of the whole pool of investments.

Further results. We construct an equilibrium in which money runs happen on the equilibrium path due to "confidence crises" that occur with sunspot probability λ. Under the assumption that confidence crises happen only if B's debt is demandable, we ask: what is the largest λ for which B still makes its debt demandable? Our model is tractable enough to admit a closed-form expression for this number. For "reasonable" parameters, we find that it is large (about 14%), suggesting that our model can plausibly explain why banks choose run-prone instruments even though doing so exposes them to costly liquidation.

We also explore three extensions. (i) We show that if B can choose its investment, its choice can be distorted toward high-liquidation-value investments, which facilitate its issuing demandable debt. (ii) We include a continuum of creditors, and show that the results of our baseline model are not driven by our assumption that there is only a single creditor. (This setup also has the attractive feature that not every withdrawal is a run.) (iii) We add random trading costs and show that our results generalize to this setup.

Layout. The rest of the Introduction includes discussions of policy, contemporary applications, empirical content, and related literature. Section 2 presents the model. Section

3 analyzes benchmarks. Section 4 includes our main results. Section 5 studies extensions. Section 6 is the conclusion.

1.1 Policy, Applications, and Empirical Content

Policy. Our analysis suggests that financial fragility may be a necessary evil given secondary-market trading frictions—money runs are the cost of the debt capacity afforded by demandable debt. However, decreasing secondary-market trading frictions can make banks less reliant on demandable debt, decreasing the likelihood of runs. Thus, we suggest that to improve bank stability, a policy maker may be better off decreasing secondary-market frictions than regulating banks directly. In contemporary markets, we think that centralized exchanges and clearing houses for bank bonds, like those for stocks, could decrease trading frictions in a way that decreases banks' reliance on overnight (effectively demandable) debt.

Unlike in Diamond and Dybvig's (1983) model of bank runs, in which suspension of convertibility restores efficiency, in our model suspension of convertibility may have an adverse effect. Since it prevents creditors from redeeming on demand to meet their liquidity needs, it leads to lower secondary-market debt prices and, hence, to constrained bank borrowing and inefficient investment.

Finally, capital requirements are a double-edged sword. They can help, by curbing banks' incentive to use too much demandable debt. But they can also hurt, by inefficiently constraining investment.

Contemporary applications. Our model does not apply only to money in the nineteenth century, when you would trade banknotes OTC to get beer from the barman. Much of the money used today is also bank debt. Today, you exchange repos in an OTC market to get liquidity from a financial counterparty. More specifically, in a repo contract, a creditor gets not only a borrower's promise to repay—its debt—but also the borrower's collateral. Even though the borrower's debt does not circulate as money—it is formally a bilateral agreement, not a tradeable instrument—its collateral does. As Gorton and Metrick (2010) put it,[3]

> [An] important feature of repos is that the...collateral can be "spent"...used as collateral in another, unrelated, transaction.... This...means that there is a money velocity associated with the collateral. In other words, the same collateral can support multiple transactions, just as one dollar of cash can lead to a multiple of demand deposits at a bank. The collateral is functioning like cash (p. 510).

[3]See also Lee (2015), Singh and Aitken (2010), and Singh (2010).

Repos are also effectively redeemable on demand, since repo positions are typically subject to daily renegotiation and possible "withdrawal."[4] Bank deposits are also private money. Debit payments and bank transfers are bilateral (OTC) transfers of bank deposits. Of course, most deposits are redeemable on demand. So are money market mutual funds, which also resemble private money. And, like the money in our model, repos, deposits, and mutual funds are vulnerable to runs (see Gorton and Metrick (2010, 2012) and Krishnamurthy, Nagel, and Orlov (2014) for repos, Iyer and Puri (2012) for deposits, and Schmidt, Timmermann, and Wermers (2016) for money market mutual funds).[5]

Empirical content. Our model is motivated by empirical observations about financial fragility and circulating bank debt. In particular, our model offers an explanation of the following facts: (i) runs on bank debt are relatively common, even when the debt is backed by collateral; (ii) runs are often precipitated by the failure of debt to circulate in secondary markets; and (iii) banks choose to borrow via demandable debt even though it exposes them to costly runs.

Our model also casts light on several other stylized facts. (i) Demandable bank instruments, such as banknotes, repos, and deposits, are more likely to serve as media of exchange than other negotiable instruments, such as bonds and shares. In the model, this is because the option to redeem on demand props up the secondary-market price of bank debt. Thus, if you hold a variety of instruments, you prefer to use demandable instruments to raise liquidity from the secondary market and to hold long-term instruments until maturity. (ii) Our model casts light on why bank debt is more likely to be demandable than corporate debt. Banks, almost by definition, have a horizon mismatch between their assets and liabilities—they perform maturity transformation. In the model, this horizon mismatch prevents you from borrowing via other instruments. Corporates are less likely to suffer from the horizon mismatch, and are therefore more likely to fund themselves with bonds or bank loans, which do not expose them to costly runs/liquidation. (iii) Our model casts light on why nineteenth-century banknotes traded at a greater discount in markets farther away from the issuing bank (see Gorton (1996)): distance from the issuer made the notes harder to redeem on demand, weakening note holders' bargaining positions in the secondary market and decreasing the price of banknotes. (iv) Our model generates runs even with a single depositor, consistent with the fact that many runs are not market-wide, but rather occur in isolation. Indeed, Krishnamurthy, Nagel, and Orlov (2014) find that repo runs occurred in

[4] In contrast to other short-term debt positions, repo positions are kept open by default, not settled and reopened. In this sense, closing a maturing repo position is like redeeming on demand, and repos are more like banknotes than like short-term bonds.

[5] This empirical work notwithstanding, some still question the importance of liquidity risk in the 2008–2009 financial crisis, stressing solvency risk. See, e.g., Thakor (2018).

relative isolation during the financial crisis.

1.2 Related Literature

We make four main contributions to the literature.

First, we offer a new rationale for demandable debt. This adds to the literature in two ways. (i) It complements the literature that shows how demandability can help to mitigate moral hazard problems (Calomiris and Kahn (1991) and Diamond and Rajan (2001a, 2001b)).[6] In particular, we show how demandability can help to increase the value of bank debt as "private money." Thus, our model connects two of the main features of bank liabilities: they circulate as money and are redeemable on demand. (ii) It provides a counterpoint to the literature that suggests that tradeability can substitute for demandability. Notably, Jacklin (1987) shows that, in Diamond and Dybvig's (1983) environment, you do not need to redeem debt on demand if you can just trade it in the secondary market.[7,8] We show that if bank debt is traded in an OTC market, like banknotes, repos, and deposits are, then demandability complements tradeability by increasing the price at which it trades.

Second, we uncover a new kind of bank run. By connecting the fragility of money to the fragility of banks, this adds both to the literature on coordination-based bank-run models following Diamond and Dybvig (1983) and to the literature on search-based money models following Kiyotaki and Wright (1989, 1993). In these money models, monetary exchange is fragile since trade is self-fulfilling. Similarly, in the bank run models, bank deposits are fragile since withdrawals are self-fulfilling. To the best of our knowledge, we are the first to show that such bank fragility follows immediately from such monetary fragility[9] and, hence,

[6]In their conclusion, Diamond and Rajan (2001a) make the link between demandability and circulating banknotes informally, saying that

> deposits are readily transferable, and liquid, because buyers of deposits have no less ability to extract payment than do sellers of deposits. Thus, the deposits can serve as bank notes or checks that circulate between depositors. This could explain the special role of banks in creating inside money (p. 425).

We make this link formally in this paper.

[7]However, Jacklin (1987) does point out that tradeable debt can have one disadvantage relative to demandable debt: investments at the initial date can be distorted in anticipation of trading later on (see also Allen and Gale (2004), Farhi, Golosov, and Tsyvinski (2009), and Kučinskas (2017)).

[8]Other papers show that there may still be a role for demandability if tradeability is limited (Allen and Gale (2004), Antinolfi and Prasad (2008), Diamond (1997), and von Thadden (1999)). In these models, banks issue demandable debt *in spite of* trade in secondary markets, e.g., to overcome trading frictions, such as limited market participation. In our model, banks issue demandable debt *because of* trade in secondary markets—the option to redeem on demand improves the terms of trade in the secondary market.

[9]A number of papers study bank money creation independently of financial fragility (e.g., Donaldson, Piacentino, and Thakor (2017), Gu, Mattesini, Monnet, and Wright (2013), and Kiyotaki and Moore (2001a)) and some others embed Diamond–Dybvig runs in economies with private money (e.g.,

coordination-based bank runs can occur even with a single depositor—i.e. without multiple depositors racing to withdraw from a common pool of assets.[10] This helps to explain how runs can occur on collateral-backed debt, complementing the existing literature (see footnote 1).

Third, we show that the need to create circulating demandable debt gives rise to a number of other banking activities. This adds to the literature on the foundations of banking, connecting pooling assets (e.g., Boyd and Prescott (1986), Diamond (1984), Diamond and Dybvig (1983), and Ramakrishnan and Thakor (1984)) with money creation (e.g., Donaldson, Piacentino, and Thakor (2017), Gu, Mattesini, Monnet, and Wright (2013), and Kiyotaki and Moore (2001a)). Notably, in contrast to papers that emphasize how pooling helps banks meet redemptions in equilibrium via diversification, we show that pooling improves creditors' option to redeem off equilibrium even absent diversification.

Fourth, our analysis of security design in the presence of an OTC secondary market adds to the literature in three ways. (i) It complements the search-based money literature which analyzes which type of asset is the socially optimal medium of exchange for trade in the secondary market (e.g., Kiyotaki and Wright (1989) and Burdett, Trejos, and Wright (2001)). We analyze which type of contract is the privately optimal circulating instrument for funding in the primary market. (ii) It extends results in the literature on corporate bonds that suggests short-maturity bonds may have high resale prices in the secondary market (Bruche and Segura (2016) and He and Milbradt (2014)). These papers restrict attention to debt contracts as in Leland and Toft (1996). We point out that with more general contracts the option to redeem on demand provides a way to prop up secondary-market prices that does not require rolling over maturing debt.[11] (iii) It provides a counterpoint to the literature that suggests that security design may prevent bank runs (e.g., Andolfatto, Nosal, and Sultanum (2017), Green and Lin (2003), and Peck and Shell (2003)). This literature suggests that if the space of securities is rich enough, then bank runs do not arise in Diamond and Dybvig's (1983) environment. Our analysis suggests that the security designs proposed in this literature may not prevent all kinds of bank runs. This is because, in our environment, it is exactly the possibility of a run, i.e. the option to redeem on demand, that makes the banknote the

Champ, Smith, and Williamson (1996) and Sanches (2015); see also Sultanum (2016)). Relatedly, Sanches (2016) argues that banks' inability to commit to redeem deposits can make private money unstable.

[10]Our focus on runs that result from dynamic coordination failures among counterparties in the secondary market complements models that focus on runs that result from dynamic coordination failures among depositors in the primary market (the dynamic analog of Diamond–Dybvig-type runs), such as He and Xiong (2012); see also Qi (1994).

[11]In an extension, Bruche and Segura (2016) do consider a version of puttable debt. However, they effectively assume it is not tradeable, which shuts down the interaction of demandability and tradeability that is critical to our results.

optimal funding instrument.

More broadly, this paper complements the related line of research that focuses on information, rather than OTC trading frictions, in secondary-market trade (Chemla and Hennessy (2014), Dang, Gorton, and Hölmstrom (2015a, 2015b), Dang, Gorton, Holmström, and Ordoñez (2017), Gorton and Ordoñez (2014), Gorton and Pennacchi (1990), Jacklin (1989), and Vanasco (2016)). This literature generally focuses on fundamental risk, and suggests that information frictions in the secondary market lead banks to do risk transformation and that this improves social efficiency. We focus on coordination risk, and suggest that OTC trading frictions in the secondary market lead banks to do liquidity transformation but that this can decrease social efficiency. That said, we abstract from information frictions only to make our novel results stark. In practice, the mechanisms in our model are likely to complement and amplify those in this literature: coordination failures are likely to be triggered by fundamental shocks and asymmetric information is likely to exacerbate trading frictions in the OTC market.

2 Model

In this section, we present the model.

2.1 Players, Dates, and Technologies

There is a single good, which is the input of production, the output of production, and the consumption good. Time is discrete and the horizon is infinite, $t \in \{0, 1, ...\}$.

There are two types of players, a borrower B and infinitely many deep-pocketed creditors C_0, C_1, ..., where C_t is "born" at Date t. Everyone is risk-neutral and there is no discounting. B is penniless but has a positive-NPV investment. The investment costs c at Date 0 and pays off $y > c$ at a random time in the future, which arrives with intensity ρ. Thus, the investment has NPV $= y - c > 0$ and expected horizon $1/\rho$. B may also liquidate the investment before it pays off; the liquidation value is $\ell < c$.

B can fund its project by borrowing from a creditor. However, there is a horizon mismatch similar to that in Diamond and Dybvig (1983): creditors may need to consume before B's investment pays off. Specifically, creditors consume only if they suffer "liquidity shocks," which arrive at independent random times with intensity θ (after which they die). Hence, a creditor's expected "liquidity horizon" is $1/\theta$.

For now, we focus on a single borrower funding a single investment with debt to a single creditor; this helps us to distinguish the forces in our model from those in the literature.[12] Later, we include multiple borrowers funding multiple investments from multiple creditors; this allows us to show how the forces in our model give rise to something that looks like real-world banking.

2.2 Borrowing Instruments

At Date 0, B borrows the investment cost c from its initial creditor C_0 via an instrument with terminal repayment $R \leq y$, made when the investment pays off, and redemption value $r \leq \ell$, made if the instrument is redeemed earlier. Creditors can exchange the instrument among themselves and B must repay whichever creditor holds it. Hence, the instrument is tradeable demandable debt, and we refer to it as a "banknote," although it also resembles a bank deposit or a repo. We let v_t denote the Date-t value of B's debt to a creditor not hit by a liquidity shock.

As benchmarks, we consider instruments that may not be tradeable (so B has to repay C_0) and/or may not be demandable, but may be "long-term" (so B makes only the terminal repayment). So, we allow B to borrow via the banknote or one of the following debt instruments: (i) non-tradeable long-term debt, which we refer to as a "loan," (ii) non-tradeable demandable debt, which we refer to as a "puttable loan"; and (iii) tradeable long-term debt, which we refer to as a "bond" (although it also resembles an equity share). These instruments are summarized in Figure 1. They constitute all of the feasible Markovian instruments in the sense that they are all transfers from B to the debtholder that can depend on the state of B's investment at Date t (but not on the date itself) and do not violate B's limited-liability constraints.

Figure 1: Debt Instruments

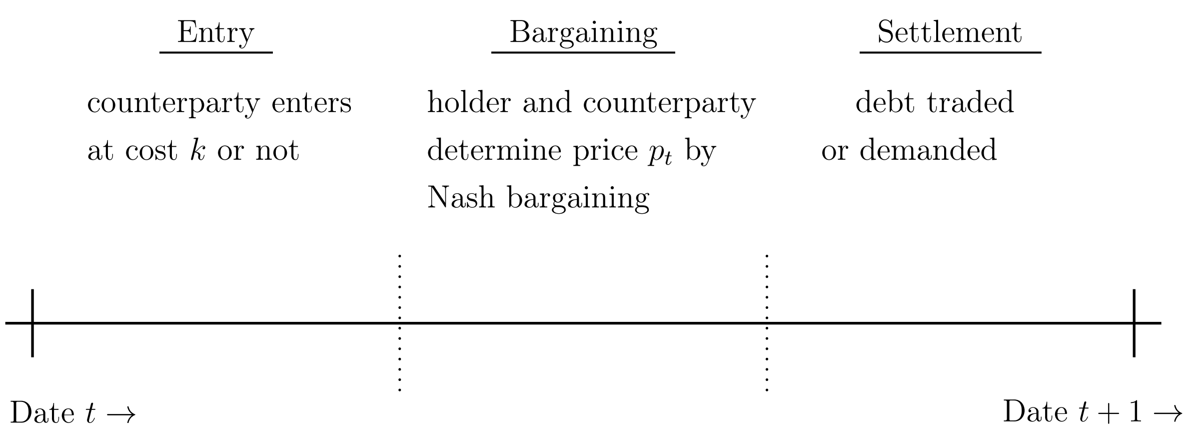
	not demandable	demandable
non-tradeable	"loan"	"puttable loan"
tradeable	"bond"	"banknote" (deposits or repos)

[12] For example, there is no coordination problem among multiple creditors (but we show there can be a different coordination problem with a single creditor) and there is no possibility to pool multiple investments (but we show a new reason to pool investments in an enriched environment (Subsection 4.4)).

2.3 Secondary Debt Market: Entry, Bargaining, and Settlement

If B has borrowed via tradeable debt, then creditors can trade it bilaterally in an OTC market. At each Date t, C_t is the single (potential) counterparty with whom the debtholder, denoted by H_t, can trade B's debt. If C_t pays an entry cost[13] k, he meets H_t. Then, C_t and H_t determine the price p_t via generalized Nash bargaining. H_t's bargaining power is denoted by η. If C_t and H_t agree on a price, then trade is settled: C_t becomes the debtholder in exchange for p_t units of the good. Otherwise, H_t retains the debt. If the debt is demandable, H_t can demand redemption from B or he can remain the debtholder at Date $t+1$. This sequence of entry, bargaining, and settlement is illustrated in Figure 2.[14] (The entry and bargaining stages are standard in the literature; the settlement stage is our addition to model demandable debt.)

FIGURE 2: SECONDARY-MARKET TRADE



[13] Historically, this "entry" cost could represent the physical cost of coming to market or, alternatively, of acquiring the expertise/technology to check for counterfeit instruments. Today, it could represent the cost of setting up a trading desk to participate in a specific market (e.g., the repo market) or, alternatively, of establishing the legal infrastructure to handle certain instruments (e.g., the GMRA master agreement for repos). More generally, it could represent any cost of searching for a counterparty as in the search money literature, of trading/transacting as in the finance literature, or of posting a vacancy as in the labor literature. All that matters for our results is that C_t bears some fixed cost to get B's debt from H_t. Further, our main results are only slightly modified if this entry cost is zero but the debtholder must pay a per-period holding cost δ as in Duffie, Gârleanu, and Pedersen (2005) (setting $\delta = \big(\rho + (1-\rho)\theta\big)k$ gives the same ex ante surplus to H_t and C_t).

[14] It may be worth emphasizing that this setup precludes the rollover strategy in which B borrows via one-period contracts at each date, borrowing enough from C_t to close its contract with H_t. Since B would have to settle first with C_t and then with H_t, this would require an additional settlement stage. That said, even if we allowed for rollover here, it would be less desirable than demandable debt as long as it had the same cost as secondary market trade. The reason is that the cost would be paid each date no matter what, instead of only when debtholders wanted to trade.

We let σ_t denote C_t's mixed strategy if H_t is hit by a liquidity shock, so $\sigma_t = 1$ means that C_t enters for sure and $\sigma_t = 0$ means that C_t does not enter. Thus, σ_t also represents the probability that H_t finds a counterparty when hit by a liquidity shock. Observe that we restrict attention to C_t's strategy given H_t is hit by a liquidity shock without loss of generality.[15]

2.4 Timeline

First, B makes C_0 a take-it-or-leave-it offer of a repayment and a redemption value, as described in Subsection 2.2 above. Then, if C_0 accepts, he becomes the initial debtholder H_1. The debtholder may redeem on demand or may trade in the secondary market, as described in Subsection 2.3 above. Formally, the extensive form is as follows.

Date 0

B offers C_0 a repayment R and a redemption value r.

If C_0 accepts, then B invests c. C_0 is the initial debtholder, $H_1 = C_0$.

Date $t > 0$

If B's investment pays off: B repays R to H_t and B consumes $y - R$.

If B's investment does not pay off: there is entry, bargaining, and settlement as described in Subsection 2.3.

If there is trade, C_t becomes the new debtholder, $H_{t+1} = C_t$.

If there is no trade, H_t either holds the debt, $H_{t+1} = H_t$, or redeems on demand, in which case B liquidates its investment, repays r to H_t, and consumes $\ell - r$.

2.5 Equilibrium

The solution concept is subgame perfect equilibrium. An equilibrium constitutes (i) the repayments R and r, (ii) the price of debt in the secondary market p_t at each date, and (iii) the entry strategy σ_t of the potential counterparty C_t[16] such that B's choice of instrument and C_t's choice to enter are sequentially rational, p_t is determined by Nash bargaining, and each player's beliefs are consistent with other players' strategies and the outcomes of Nash bargaining.

For most of the paper, we focus on stationary equilibria, i.e. $\sigma_t \equiv \sigma$ and $p_t \equiv p$.

[15]The reason that this is without loss of generality is that C_t would never enter if H_t were shocked: if H_t is not shocked, H_t and C_t are identical and there are no gains from trade, so it is never worth it to pay the cost k for the opportunity to trade.

[16]Formally, we could also include the debtholder's choice whether to redeem on demand. We omit this, however, and just assume that the debtholder redeems on demand whenever he is hit by a liquidity shock and does not trade the debt. This assumption is just for simplicity; it does not affect the equilibrium.

3 Benchmarks

To begin, we consider three benchmark instruments, the loan, the puttable loan, and the bond. We verify two results in the literature in our environment: (i) demandability can increase debt capacity as in Calomiris and Kahn (1991) and (ii) tradeability can substitute for demandability as in Jacklin (1987).

3.1 Loan

First, we consider a loan, i.e. non-tradeable long-term debt. At Date t, the value v_t of the loan with face value R can be written recursively:

$$v_t = \rho R + (1-\rho)(1-\theta)v_{t+1}. \tag{1}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the loan is neither tradeable nor demandable, H_t gets zero. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value v_{t+1} at Date t since there is no discounting.[17] By stationarity ($v_t = v_{t+1} \equiv v$), equation (1) gives

$$v = \frac{\rho R}{\rho + (1-\rho)\theta}. \tag{2}$$

Even though B will always repay eventually, the loan's value v is less than its face value R. The loan is discounted because, without the option to demand debt or trade it, B gets nothing in the event of a liquidity shock. Hence, the discount vanishes as shocks become unlikely, $v \to R$ as $\theta \to 0$. For $\theta > 0$, demandability and tradeability can help to reduce the discount, as we see next.

[17] Formally, the value of holding B's debt is the Date-t *expected* value of B's debt at Date $t+1$, i.e. we should write $\mathbb{E}_t[v_{t+1}]$ instead of v_{t+1}. For now, we focus on deterministic equilibria. Thus, this difference is immaterial and we omit the expectation operator for simplicity. (In Subsection 4.3, we do keep track of the expectation operator.)

3.2 Puttable Loan

Now we consider a puttable loan, i.e. non-tradeable demandable debt. At Date t, the value v_t of the puttable loan can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta r + (1-\theta) v_{t+1}\Big). \tag{3}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the loan is demandable, but not tradeable, H_t redeems on demand and gets r. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value v_{t+1} at Date t since there is no discounting. By stationarity ($v_t = v_{t+1} \equiv v$), equation (3) gives

$$v = \frac{\rho R + (1-\rho)\theta r}{\rho + (1-\rho)\theta}. \tag{4}$$

We now compare the puttable loan's debt capacity with the loan's, where "debt capacity" refers to the maximum B can borrow given limited liability. I.e. we compare equation (4) with $R = y$ and $r = \ell$ and equation (2) with $R = y$:

Proposition 1. (Benchmark: benefit of demandability.) *If*

$$\frac{\rho y}{\rho + (1-\rho)\theta} < c \leq \frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{5}$$

then B can fund itself with a puttable loan but not with a loan.

The analysis so far already points to one rationale for demandable debt. As in Calomiris and Kahn (1991), the option to liquidate insures C_0 against bad outcomes, making him more willing to lend.[18] Thus, by issuing demandable debt, B expands its debt capacity.

3.3 Bond

Now we consider a bond, i.e. tradeable long-term debt.[19] (This instrument can also represent an equity claim; debt and equity have equivalent payoffs, since the terminal payoff y is

[18]In Calomiris and Kahn (1991), "bad outcomes" are associated with moral hazard problems, rather than liquidity shocks.

[19]Kiyotaki and Moore (2001a, 2001b, 2002, 2005) also differentiate between tradeable and non-tradeable debt, taking tradeability as exogenous. Donaldson and Micheler (2016) analyze a model in which tradeability is chosen by the borrower.

deterministic.) At Date t, the value v_t of the bond can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta\sigma_t p_t + (1-\theta)v_{t+1}\Big). \tag{6}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the bond is tradeable, but not demandable, H_t gets p_t if he finds a counterparty, which happens with probability σ_t, and nothing otherwise. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains B's debt at Date $t+1$, which has value v_{t+1} at Date t since there is no discounting.

To solve for the value v_t, we must first find the secondary-market price of the bond p_t.

Lemma 1. *The secondary-market price of the bond is $p_t = \eta v_t$.*

The bond price splits the gains from trade between H_t and C_t in proportions η and $1-\eta$. Since H_t has value zero in this case (H_t dies at the end of the period and the bond is not demandable), the gains from trade are just the value v_t of the bond to the new debtholder C_t.

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$) and the preceding lemma ($p_t \equiv p \equiv \eta v$), equation (6) gives

$$v = \frac{\rho R}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{7}$$

We now compare bond's debt capacity (equation (7) with $R = y$ and $\sigma = 1$)[20] to the puttable loan's (equation (4) with $R = y$ and $r = \ell$):

Proposition 2. (Benchmark: tradeability substitutes demandability.) *Suppose the bond circulates in equilibrium ($\sigma = 1$).*[21] *If*

$$\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta} < c \leq \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}, \tag{8}$$

then B can fund itself with a bond but not with a puttable loan (or a loan).

If the bond circulates, B can borrow against the full value y whenever trading frictions vanish (in the sense that H_t gets the bargaining power). I.e. if $\sigma = 1$, then there is no role

[20] The debt capacity of a tradeable instrument refers to the maximum B can borrow *if* it circulates, or $\sigma = 1$. Thus, since σ is chosen by C_t, the debt capacity is an upper bound on what B can borrow. I.e. the condition that the debt capacity exceeds c is necessary but not sufficient for B to invest (see, however, footnotes 21 and 22).

[21] As we will see below (setting $r = 0$ in equation (13)), there is an equilibrium in which the bond circulates as long as C_t's entry cost k is sufficiently small.

for demandability whenever $\eta \to 1$. Hence, the analysis so far supports Jacklin's (1987) intuition that tradeability substitutes for demandability. If C_0 is hit by a liquidity shock, he can trade B's debt in the market, rather than die with it. In other words, like the option to demand, the option to trade insures C_0 against bad outcomes, making him more willing to lend. Indeed, absent trading frictions ($\eta \to 1$), B can expand its debt capacity more by issuing tradeable debt (a bond) than by issuing demandable debt (a puttable loan). However, we will see next that with trading frictions ($\eta < 1$), there is a role for demandability, even if debt is never demanded in equilibrium (Proposition 3).

4 Banknote and Banking

In this section, we analyze the banknote and present our main results.

4.1 Bright Side of Demandable Debt

Now we consider a banknote, i.e. tradeable, demandable debt. At Date t, the value v_t of the banknote can be written recursively:

$$v_t = \rho R + (1-\rho)\Big(\theta\big(\,\sigma_t p_t + (1-\sigma_t)r\,\big) + (1-\theta)v_{t+1}\Big). \tag{9}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R. With probability $(1-\rho)\theta$, B's investment does not pay off and the debtholder H_t is hit by a liquidity shock. Since the banknote is both tradeable and demandable, H_t gets p_t if he finds a counterparty, which happens with probability σ_t, and otherwise redeems on demand and gets r. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. H_t retains the banknote at Date $t+1$, which has value v_{t+1} at Date t since there is no discounting.

To solve for the value v_t, we must first give the secondary-market price of the banknote p_t.

Lemma 2. *The secondary-market price of the banknote is* $p_t = \eta v_t + (1-\eta)r$.

The price of the banknote splits the gains between H_t and C_t in proportions η and $1-\eta$. Since H_t has value r (H_t redeems on demand and gets r if he does not trade with C_t), the gains from trade are $v_t - r$, the value to the new debtholder C_t minus the value to the current debtholder H_t. The price that splits these gains is $p_t = r + \eta(v_t - r) = \eta v_t + (1-\eta)r$.

By stationarity ($v_t = v_{t+1} \equiv v$ and $\sigma_t \equiv \sigma$) and the preceding lemma ($p_t \equiv p = \eta v + (1-\eta)r$), equation (9) gives

$$v = \frac{\rho R + (1-\rho)\theta\big(1-\eta\sigma\big)r}{\rho + (1-\rho)\theta\big(1-\eta\sigma\big)}. \tag{10}$$

We now compare the banknote's debt capacity (v with $R = y$, $r = \ell$, and $\sigma = 1$) to the benchmark instruments' (Section 3). We find that B can borrow more via a banknote than via any other instrument.

PROPOSITION 3. (BRIGHT SIDE.) *Suppose the banknote circulates ($\sigma = 1$).*[22] *If*

$$\max\left\{\frac{\rho y + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \frac{\rho y}{\rho + (1-\rho)\theta(1-\eta)}\right\} < c \le \frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)}, \tag{11}$$

then B can fund itself only with the banknote.

Unlike the puttable loan, the banknote need not be redeemed in equilibrium. Like the bond, it can circulate in the secondary market until maturity. But it is still more valuable than the bond. The reason is that the option to redeem the banknote on demand puts the debtholder in a strong bargaining position in the secondary market, increasing its price. Thus, given secondary market trading frictions ($\eta < 1$), demandability *complements* tradability: your option to demand debt increases the price you trade at. This high price leads to a high debt capacity: in anticipation of being able to sell at a high price in the secondary market, C_0 is willing to pay a high price in the primary market.[23]

What kind of borrower needs to issue the banknote? To answer, we rewrite the condition of Proposition 3. From the left-most term of equation (11):

$$\frac{1}{\rho} > \frac{1}{\theta} \cdot \frac{y-c}{(1-\rho)(1-\eta)}. \tag{12}$$

This says that creditors' expected liquidity horizon $1/\theta$ is small relative to B's expected investment horizon $1/\rho$. Hence, B's debt is a kind of inside money, since a creditor generally does not hold it for its entire maturity; rather he holds it for a short time and then uses it to get liquidity from another creditor—as Kiyotaki and Moore (2001a) put it, "[w]henever

[22] We will see below (equation (13)) that there is an equilibrium in which the banknote circulates as long as C_t's entry cost k is sufficiently small.

[23] Relatedly, He and Milbradt (2014) and Bruche and Segura (2016)) argue that short maturity has the benefit of high secondary market prices, but the cost of frequent debt issuances. We show that if demandable debt is allowed, the benefit can come without the cost, since demandable debt need not ever be redeemed. Demandable debt thus resembles perpetual debt. However, we will show below it comes with another cost, a new kind of liquidation risk.

paper circulates as a means of short-term saving (liquidity), it can properly be considered as money, or a medium of exchange, because agents hold it not for its maturity value but for its exchange value" (p. 1). Moreover, it implies that B intermediates between short-horizon creditors and a long-horizon investment. Hence, B is starting to resemble a bank, as maturity transformation is one of banks' defining features. But this is just the first step in our argument that B is a bank. Below, we will see that B shares a number of banks' other defining features, all of which stem from B's need to create valuable private money (Subsection 4.4).

4.2 Money Runs

Having established how a banknote helps B raise funds in the primary market, we now turn to how it trades in the secondary market, and whether it could be in fact redeemed early. In other words, does the banknote always circulate ($\sigma = 1$), as we assumed above? To answer, we assume that B has issued a banknote at Date 0 with terminal repayment R and redemption value r, and we look at the equilibria of the subgames for $t > 0$. (We determine R and r in equilibrium in Proposition 5.)

First, observe that B's banknote circulates as long as $\sigma_t = 1$ is a best response to the belief that $C_{t'}$ plays $\sigma_{t'} = 1$ for all $t' > t$. This is the case as long as C_t is willing to pay the entry cost k to gain the surplus $v - p$ given $\sigma = 1$, or

$$k \;\leq\; v - p\,\Big|_{\sigma=1} = \frac{\rho(1-\eta)(R-r)}{\rho + (1-\rho)\theta(1-\eta)}, \tag{13}$$

having substituted in from Lemma 2 and equation (10).

But there may also be another equilibrium in which B's banknote does not circulate. B's banknote does not circulate as long as $\sigma_t = 0$ is a best response to the belief that $C_{t'}$ plays $\sigma_{t'} = 0$ for all $t' > t$. This is the case as long as C_t is *not* willing to pay the entry cost k to gain the surplus $v - p$ given $\sigma = 0$, or

$$k \geq v - p\,\Big|_{\sigma=0} = \frac{\rho(1-\eta)(R-r)}{\rho + (1-\rho)\theta}, \tag{14}$$

again having substituted in from Lemma 2 and equation (10).[24]

Proposition 4. (Money runs.) *Suppose that B borrows via a banknote with terminal*

[24]For fixed r, this "bad" equilibrium arises only for sufficiently high k. However, for endogenous r, it can arise for arbitrarily small $k > 0$; see Subsection 4.4. See Rocheteau and Wright (2013) for a model in which multiple (non-steady state) equilibria arise in a decentralized market without a fixed cost.

repayment R and redemption value r. If the entry cost k is such that

$$\frac{\rho(1-\eta)(R-r)}{\rho+(1-\rho)\theta} \leq k \leq \frac{\rho(1-\eta)(R-r)}{\rho+(1-\rho)\theta(1-\eta)}, \tag{15}$$

then the $t>0$ subgame has both an equilibrium in which B's debt circulates ($\sigma=1$) and there is no early liquidation and an equilibrium in which B's debt does not circulate ($\sigma=0$) and there is early liquidation. There is also a mixed equilibrium, with

$$\sigma = \frac{1}{\eta}\left(1-\frac{\rho}{(1-\rho)\theta k}\Big((1-\eta)\big(R-r\big)-k\Big)\right). \tag{16}$$

If a counterparty C_t doubts future liquidity, i.e. he doubts that he will find a counterparty in the future, then C_t will not enter. As a result, the debtholder H_t indeed will not find a counterparty. There is a self-fulfilling dry-up of secondary-market liquidity. With demandable debt, this has severe real effects: unable to trade, H_t redeems his debt on demand, leading to the costly liquidation of B's investment. In other words, a change in just the beliefs about future liquidity leads to the failure of B's debt as a medium of exchange in the secondary market—the failure of B's debt as money. As a result, there is sudden withdrawal of liquidity from B, i.e. a bank run, or a *money run.*

Corollary 1. *Suppose k satisfies condition (15). If C_t's beliefs change from $\sigma_{t'}=1$ to $\sigma_{t'}=0$ for $t'>t$, the debtholder H_t "runs" on B, i.e. H_t unexpectedly demands redemption of his debt, forcing B to liquidate its investment.*

The literature has stressed bank failures resulting from shocks to fundamentals (e.g., Allen and Gale (1998) and Gorton (1988)) or beliefs about primary market withdrawals (Diamond and Dybvig (1983)). Friedman and Schwartz (2008) emphasize that such bank failures, whatever their root cause, disrupt economic activity because banks create money—e.g., they issue banknotes—which facilitates trade. Our model also connects bank failure with money creation. But the chain of causation goes in the opposite direction: the banknote is redeemed only because it fails to circulate.

In our model, financial fragility is a necessary evil. It is necessary because B must issue a fragile instrument—the banknote—to fund itself (Proposition 3). And it is evil because money runs lead to inefficient liquidation. This contrasts with the literature on the necessity of financial fragility, which stresses its virtue, not evil (Allen and Gale (1998), Diamond and Rajan (2001a, 2001b)).

Although financial fragility is necessary in our model, it can still be excessive. To see why, first observe that increasing the redemption value r makes runs "more likely": high r

puts H_t in a strong bargaining position, increasing the price C_t pays. This makes it less attractive for him to enter. And if C_t does not enter, H_t is unable to trade and must redeem early—must run.

COROLLARY 2. *Increasing the redemption value r makes the banknote less likely to circulate in the following senses:*

(i) each counterparty C_t enters only for lower entry cost k (given the strategy of other counterparties);

(ii) $\sigma = 1$ is an equilibrium of the $t > 0$ subgame only for lower k;

(iii) $\sigma = 0$ is not an equilibrium of the $t > 0$ subgame for lower k.

Hence, demandability cuts both ways. It is both the thing that allows B to fund itself and the thing that exposes B to runs. It increases B's debt capacity, since it props up the price of B's debt (Proposition 3). But it also increases B's liquidation risk, since it makes C_t reluctant to enter.

Does B internalize the full cost of liquidation risk? The next result says that the answer is no.

LEMMA 3. *Suppose that the probability that each counterparty enters is an increasing function f of $(R - r)$.*[25] *As long as the derivative f' is not too large, B sets the maximum redemption value, $r = \ell$.*

Intuitively, there is a benefit to B of increasing the redemption value r: C_0 requires less compensation for the risk of having to sell at a discount in the secondary market. This benefit is a cost to C_0's future counterparty, who pays a high price for the banknote. But he is not there at Date 0, when B and C_0 are bargaining. Hence B does not fully internalize this cost, and continues to increase r even when it has no social benefit.

[25] To be clear, for this result, we consider B's best response to counterparties' strategies $\sigma = f(R - r)$, where f depends on $R - r$, as motivated by C_t's entry condition (equation (40)). For now, we do not require these strategies to be consistent, as in equilibrium. However, we show in Subsection 5.3 below that if the entry cost is a random variable $\tilde{k}_t$, such strategies arise in equilibrium (and modifying the distribution of $\tilde{k}_t$ gives us some freedom to generate different functions f).

The direct effect of increasing r is to make C_t less likely to enter, as stated in Corollary 2. We capture this by assuming that $f' > 0$. But we should point out that increasing r can also have an indirect effect, since it can change counterparties' equilibrium strategies. Indeed, in some circumstances this indirect effect can be important. E.g., in the mixed equilibrium in Proposition 4, it opposes the direct effect, and increasing r actually makes C_t more likely to enter. We could capture this by assuming that $f' < 0$, but we do not focus on this case, since the trade-off would be assumed away: increasing r would not only increase the price of the banknote, but also make it more likely to circulate.

4.3 Equilibrium Runs

We now turn to characterizing an equilibrium in which B borrows via a banknote and money runs arise on the equilibrium path. To do this, we introduce a "sunspot" coordination variable at each date, $s_t \in \{0, 1\}$. We will interpret $s_t = 1$ as "normal times" and $s_t = 0$ as a "confidence crisis," since the sunspot does not affect economic fundamentals, but serves only as a way to coordinate beliefs. We assume that $s_0 = 1$, that $\mathbb{P}[s_{t+1} = 0 \,|\, s_t = 1] =: \lambda$, and that $\mathbb{P}[s_{t+1} = 0 \,|\, s_t = 0] = 1$, where we think about λ as a small number. In words: the economy starts in normal times and a permanent confidence crisis occurs randomly with small probability λ.

We now look for a Markov equilibrium, i.e. an equilibrium in which the sunspot (rather than the whole history) is a sufficient statistic for C_t's action:

$$\sigma_t = \begin{cases} \sigma^1 & \text{if } s_t = 1, \\ \sigma^0 & \text{if } s_t = 0. \end{cases} \tag{17}$$

We can now write the banknote's value v as a function of s_t (cf. the analogous equation for the stationary case in equation (9)):

$$v^0 = \rho R + (1-\rho)\bigg(\theta\Big(\sigma^0 p^0 + \big(1-\sigma^0\big)r\Big) + (1-\theta)v^0\bigg), \tag{18}$$

$$v^1 = \rho R + (1-\rho)\bigg(\theta\Big(\sigma^1 p^1 + \big(1-\sigma^1\big)r\Big) + (1-\theta)\Big(\lambda v^0 + (1-\lambda)v^1\Big)\bigg). \tag{19}$$

The next proposition characterizes an equilibrium in which the "confidence crisis" induces a money run.

Proposition 5. (Equilibrium with sunspot runs.) *Suppose that the condition in equation (11) is satisfied strictly. As long as λ is sufficiently small, there exists k such that B can fund its investment only with tradeable, demandable debt (a banknote), even though it admits a money run when $s_t = 0$. Specifically, C_t plays $\sigma_t = s_t$, and the value of the banknote when $s_t = 0$ is*

$$v^0 = \frac{\rho R + (1-\rho)\theta\ell}{\rho + (1-\rho)\theta}, \tag{20}$$

the value of the banknote when $s_t = 1$ is

$$v_1 = \frac{\Big(\rho + (1-\rho)\big(\lambda(1+\theta\eta) + (1-\lambda)\theta\big)\Big)c - (1-\rho)\lambda\theta\eta\ell}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}, \tag{21}$$

the repayment R is

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - \ell\Big), \quad (22)$$

and the redemption value is $r = \ell$.

With these closed-form expressions, it is easy to see how the price of debt depends on parameters.

Corollary 3. (Comparative statics.) *The (net) interest rate $(R-c)/c$ is*

(i) decreasing in the liquidation value ℓ;

(ii) decreasing in debtholders' bargaining power η;

(iii) decreasing in creditors' liquidity horizon $1/\theta$;

(iv) increasing in the probability of a confidence crisis λ;

(v) increasing in the investment size c;

(vi) increasing in the investment horizon/expected maturity $1/\rho$. Moreover, the term structure is upward sloping, in the sense that the yield[26] *$\rho(R-c)/c$ is also increasing in $1/\rho$.*

In our model, the interest rate is compensation for liquidity risk. The results (i)–(iv) capture that increasing ℓ and η decrease liquidity risk and increasing θ and λ increase it. (v) says that bigger investments are effectively riskier (all else equal). The reason is that, for fixed liquidation value ℓ, they are liquidated at a larger discount in a confidence crisis. (vi) says that longer maturity investments demand not only higher repayments, but also higher per-period interest rates, even though there is no discounting in preferences. The reason is that as maturity increases both the probability that C_0 has to trade at a discount before maturity *and* the size of the discount he trades at increase. So illiquidity in the OTC market generates the term structure.[27]

[26] This uncompounded yield is approximately equal to the continuously compounded yield, which you might be more used to. For a zero-coupon instrument:

$$\text{continuously compounded yield} \equiv \rho \log \frac{R}{c} = \rho \log\left(1 + \frac{R-c}{c}\right) \approx \rho \frac{R-c}{c}, \quad (23)$$

for small $(R-c)/c$ (given the Taylor expansion of $\log(1+x)$).

[27] See Kozlowski (2017) for a macroeconomic model in which trading frictions generate the yield curve.

A word on welfare and a numerical example. Given its multiple equilibria, our model does not admit a general welfare analysis. To speak to welfare, we make the following assumption, motivated by the idea that confidence crises are likely only if there is the risk of early redemption: confidence crises can occur if B borrows via the banknote, but not if B borrows via the bond. We ask: if both the banknote and the bond are feasible, how small does the probability λ of a confidence crisis have to be for B to prefer the banknote?

PROPOSITION 6. (CONFIDENCE CRISIS PROBABILITY.) *Suppose confidence crises can occur only if B borrows via the banknote. If the bond is feasible, B still borrows via the banknote whenever the probability λ of a confidence crisis is below the threshold λ^*,*

$$\lambda^* = \frac{\rho\big(\rho + (1-\rho)\theta\big)(1-\eta)\ell}{\rho\big(y-\ell\big) + \big(\rho + (1-\rho)\theta\big)(1-\eta)(\rho\ell - c)}, \tag{24}$$

and borrows via the bond otherwise.

For example, if $\theta = 1/4$, $\rho = 1/10$, $y = 20$, $c = 10$, $\ell = 8$, and $\eta = 3/4$, B chooses the banknote whenever $\lambda \leq \lambda^* \approx 14.4\%$.[28] This points to a potentially attractive feature of our model: unlike in many quantitative bank run models, a borrower chooses the run-prone instrument for "reasonable" parameters even when the probability of a run is relatively high. This seems consistent with banks' behavior historically. In the nineteenth-century US, banks issued banknotes despite ubiquitous runs.

4.4 Banking

We now suppose that the horizon mismatch (equation (11)) is so severe that the borrower cannot raise c to fund its investment, even via a banknote. In this case, direct finance is not possible. But perhaps a form of intermediated finance is?

To address this question, we now consider N parallel versions of our baseline model: N identical borrowers $\mathrm{B}^1, ..., \mathrm{B}^N$ can do parallel investments at Date 0 and N identical creditors $\mathrm{C}^1_t, ..., \mathrm{C}^N_t$ can enter parallel markets at each Date $t > 0$. At Date 0, the borrowers can issue mutualized instruments, backed by the whole pool of their investments. Redeeming creditors are paid in full until resources are exhausted, when they are paid pro rata. At each subsequent date, each version of the model proceeds exactly as in the baseline model,

[28] We think about these as annual numbers. $\theta = 1/4$, the number used in Ennis and Keister (2003), implies creditors suffer liquidity shocks on average once every four years. $\rho = 1/10$ implies the investment is long-term, taking ten years to complete on average. Given this maturity, $y = 20$ and $c = 10$ imply the investment has annual return of 7.2%. $\ell = 8$ implies the investment has a 20% liquidation discount relative to its book value. $\eta = 3/4$ implies that debtholders get most of the surplus, but far from all of it; this is intended to capture some degree of competition among counterparties.

as described in Section 2. Note that we assume that the parallel versions of the model are identical in every state, i.e. investments/liquidity shocks are perfectly correlated across borrowers/creditors. Thus, there are no diversification benefits from pooling deposits/loans as in Diamond and Dybvig (1983)/Diamond (1984).

Even absent diversification, the borrowers can benefit from pooling their investments. They can issue N banknotes, each backed by the entire pool and thereby reuse the investments' liquidation values. They cannot honor all the banknotes at once in equilibrium, but they do not have to: the debtholders' option to redeem them off equilibrium is enough to increase the secondary market price and hence the borrowers' debt capacity. Not only can they fund investments that they could not have funded via banknotes, they can fund all (and only) investments with positive surplus:

Proposition 7. (Banking.) *Suppose*

$$N \geq \frac{1}{\ell}\left(y - \frac{\rho + (1-\rho)(1-\eta)}{\rho(1-\eta)}k\right). \tag{25}$$

There is an equilibrium in which borrowers successfully fund all investments, raising c by issuing a banknote to each of the Date-0 creditors, if and only if the investments have positive total surplus, i.e. the NPV is higher than the total expected entry costs,[29] *or*

$$y - c \geq \frac{(1-\rho)\theta}{\rho}k. \tag{27}$$

Intermediated finance, in which the borrowers get together to mutualize their assets and liabilities, helps because the debt capacity of the N borrowers collectively is more than N times the debt capacity of any borrower individually. This is because the redemption value r of a banknote is no longer bounded by the liquidation value ℓ of a single investment, but by the liquidation value $N\ell$ of the pool. Hence, as long as N is sufficiently large (equation (25)), r is limited only by creditors' entry condition ($k \leq v - p$), which can be re-written to give an upper bound $r^{\max}$ on r:

$$r \leq r^{\max} := R - \frac{\rho + (1-\rho)\theta(1-\eta\sigma)}{\rho(1-\eta)}k. \tag{28}$$

[29]The expression for the expected entry costs can be understood as follows: from Date 1 onward, creditors pay k at Date t if debtholders are shocked while investments are still underway, which occurs with probability $(1-\rho)^t\theta$. Hence,

$$\text{total expected entry costs } = \sum_{t=1}^{\infty}(1-\rho)^t\theta k = \frac{(1-\rho)\theta}{\rho}k. \tag{26}$$

Thus, to find the debt capacity of the banknote, we substitute $r = r^{\max}$, $R = y$, and $\sigma = 1$ into the value of the banknote (equation (10)). It turns out that the borrowers can undertake investments if and only if they have positive total surplus (as per equation (27)).

To fund these investments, the borrowers must set r so high that counterparties are indifferent between entering and staying out. This makes them especially susceptible to runs, since an arbitrarily small change in a counterparty's belief about others' strategies makes him stay out, leading to a money run. Moreover, unlike in the baseline model, runs can occur no matter how small the entry cost k is. If k is small, the borrowers make r high, so counterparties are still indifferent between entering and staying out (cf. equation (14)).

And now a money run has severe consequences. As in a real-world bank run, there is mass liquidation: with $r > \ell$, multiple investments need to be liquidated to redeem each banknote. In addition to this fragility, the coalition of borrowers has other defining features of actual banks. These features, which we now enumerate, lead us to refer to the coalition of borrowers as the "bank."

1. **Liquidity tranformation.** The bank funds illiquid assets (non-tradeable investments that are costly to liquidate early) with liquid liabilities (circulating demandable debt, where the discount for early redemption is small if entry costs are small, i.e. $R - r \to 0$, as $k \to 0$).

    - Issuing liquid (tradeable) liabilities gives creditors insurance against liquidity shocks.

2. **Maturity transformation.** The bank funds long-term investments with short-term (demandable) liabilities.

    - Issuing demandable liabilities allows creditors to trade at a high price given liquidity shocks.

3. **Asset pooling.** The bank pools borrowers' investments, reusing their liquidation value to back demandable debt.

    - Issuing debt backed by a pool of assets gives creditors a high redemption value.

4. **Dispersed depositors (creditors).** The bank borrows from a large number of dispersed creditors.

    - Issuing debt to many creditors gives them the option to redeem against the same assets (hence dispersed creditors are necessary for asset pooling to help).

5. **Fragility.** The bank borrows via debt that is susceptible to runs, and runs force early liquidation of multiple investments.

    - Issuing run-prone debt, i.e. demandable debt with high redemption value, is necessary to make the secondary market price high enough that the bank can fund efficient investments.

5 Extensions

In this section, we analyze extensions.

5.1 Asset Choice

What if B chooses the type of its investment before borrowing from C_0? Do frictions in the secondary market distort its choice? Yes, toward high-liquidation-value investments:

PROPOSITION 8. (EXCESSIVE LIQUIDITY.) *Suppose that B can choose between an investment with payoff y and liquidation value ℓ and another investment that is otherwise identical but has lower payoff $y' < y$ and higher liquidation value ℓ', where*

$$\ell' > \ell + \frac{\rho}{(1-\rho)\theta(1-\eta)}\big(y - y'\big). \tag{29}$$

There exists an investment cost c such that in any equilibrium in which investment occurs, B chooses the low-NPV, high-liquidation-value investment (y', ℓ').

Intuitively, with a high-liquidation-value investment, B can issue a high-redemption-value banknote and borrow more. Thus, to make its debt money-like, B chooses to increase its liquidation value even at the expense of NPV.[30]

[30]This idea also speaks to intermediation: if B has an investment but no liquid assets, then a bank can step in to "create liquid paper," as in Subsection 4.4. I.e. B borrows from the bank via a loan and the bank borrows from C_0 via a banknote backed by its own assets, e.g., its pool of investments. Even though these assets create no extra real value, the option to redeem them on demand puts debtholders in a strong bargaining position in the secondary market, making C_0 willing to lend. This creates real value by allowing B to fund its investment. The idea that banks emerge because their assets allow them to issue private circulating debt resonates with Kiyotaki and Moore (2001a), who say

> the trick would be to make a profit by buying blue paper [i.e. illiquid debt] and selling red [liquid debt]! Imagine...offering to lend to private people whose IOUs are illiquid. Since their paper is illiquid, blue, they have to pay you a relatively high rate of interest. Meantime, you raise funds by taking in deposits....You are making a profit, merely by sitting there! Do you know what you are? You are a bank. You effectively transform blue paper into red (p. 23).

5.2 Partial Rollover

We now turn to a version of the model in which many debtholders and many counterparties can meet in a single (decentralized) market at each date. We assume that liquidity shocks are independent across debtholders, so there is no aggregate risk. But runs can still occur following dry-ups of secondary market liquidity. This affirms that our results are driven just by coordination, not by aggregate liquidity risk. Further, we assume that B can risklessly roll over its debt at each date. But runs can still occur. This distinguishes our run risk from rollover risk.[31] Moreover, unlike in the baseline model, not every withdrawal is a run. Rather, some debtholders demand redemption at each date.

Here we do not model funding/investment, but focus on secondary market circulation and rollover, assuming that the banknotes are held by a unit continuum of debtholders. To model rollover, we assume that B matches with new creditors at the beginning of each date, raising r from each of them via new banknotes with face value R and redemption value r. Note that the amount raised and the redemption value coincide; this keeps the model stationary. And note further that B must decide how much to raise at the beginning of the date; this makes runs possible.

In the secondary market, there is a "large" continuum of counterparties at each date who can enter at cost k. But, unlike in Subsection 4.4, in which counterparties trade in parallel markets, they are matched with debtholders in a single market via a homogenous matching technology, as in the search-and-matching literature. σ_t denotes the probability that a debtholder is matched with a counterparty, as in the baseline model, except here it depends on the number of counterparties that enter. Given there is now a continuum of traders, there is no aggregate risk: a constant fraction θ of debtholders is shocked at each date and a fraction σ_t of them is matched with counterparties. These debtholders trade in the secondary market. The remaining $\theta(1-\sigma_t)$ debtholders redeem for r. We assume that B's rollover strategy is determined so as to meet these redemptions. I.e. it raises exactly $\theta(1-\sigma_t)r$ at the beginning of the date. This ensures that the mass of debtholders is constant.

The next result says that this set-up has multiple steady states. Indeed, there is a "good equilibrium," in which many counterparties enter and few debtholders are left unmatched. In this equilibrium, there are relatively few withdrawals at each date, so B chooses its rollover strategy to raise a relatively small amount of liquidity. But there is also a "bad equilibrium," in which few counterparties enter and many debtholders are left unmatched. In this equi-

[31] We use "run risk" to mean the risk of an unexpectedly large number of withdrawals. In contrast, we use "rollover risk" to mean the risk that B attempts to raise new debt and fails. Below, we assume B can roll over costlessly—there is no rollover risk—but B cannot go back to the market to meet a large number of withdrawals without some delay—there is run risk.

librium, there are more withdrawals at each date, so B has to choose a rollover strategy to raise more liquidity. Thus, a change in beliefs can lead to a money run: if counterparties believe that few other counterparties will enter in the future, then few counterparties will enter today, leading to an unexpectedly high number of withdrawals—a money run forcing liquidation.

PROPOSITION 9. (MONEY RUNS WITH PARTIAL ROLLOVER.) *Let the matching technology be given by $\sigma = \mu\sqrt{q}$, where q is the number of counterparties that enter and $\mu > 0$ is a parameter. Suppose that B borrows via banknotes from a continuum of creditors. The $t > 0$ subgame has two stationary equilibria, one in which many counterparties enter,*

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) + \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_+ \quad (30)$$

—banknotes are liquid—and another in which few counterparties enter

$$\sigma = \frac{k\big(\rho + (1-\rho)\theta\big) - \sqrt{k^2\big(\rho + (1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2k(1-\rho)\theta\eta} =: \sigma_- \quad (31)$$

—banknotes are illiquid—as long as σ_+ and σ_- above are well-defined probabilities.

This result implies that money runs can occur even with no aggregate risk, no rollover risk, and no sequential-service constraint. This affirms that money runs result only from intertemporal coordination in the secondary market, and helps distinguish our model of bank fragility from models of rollover risk (e.g., He and Xiong (2012), Acharya, Gale, and Yorulmazer (2011)) and of bank runs (Diamond and Dybvig (1983)).

5.3 Random Entry Costs

Now we modify the baseline model so that C_t's entry cost is a random variable $\tilde{k}_t \sim \mathrm{U}\left[0, \bar{k}\right]$. We find that the $t > 0$ subgame has multiple symmetric cut-off strategy equilibria.

PROPOSITION 10. (CUT-OFF EQUILIBRIA.) *Define*

$$k^{\pm} = \frac{\bar{k}}{2(1-\rho)\theta\eta}\left(\rho + (1-\rho)\theta \pm \sqrt{\big(\rho + (1-\rho)\theta\big)^2 - \frac{4\rho(1-\rho)\theta\eta(1-\eta)\big(R-r\big)}{\bar{k}}}\right). \quad (32)$$

If $k^{\pm} \in [0,1]$, then the $t > 0$ subgame has equilibria in which C_t enters whenever his entry cost is below k^* for $k^* = k^-$ and $k^* = k^+$.

Given $\tilde{k}_t \sim \mathrm{U}\left[0, \bar{k}\right]$, the probability a creditor enters is $k^*/\bar{k}$. In Lemma 3, we assume this probability is an increasing function f of $R - r$. From equation (32), we see that this assumption holds in the k^--equilibrium. Hence, this extension gives an equilibrium foundation for our assumption. (The assumption does not hold in the k^+ equilibrium. Although f is still a function of $R - r$, it is decreasing, as in the mixed equilibrium; cf. the discussion in footnote 25.)

6 Conclusion

What is a bank? A bank is something that creates money, i.e. debt that facilitates trade in decentralized markets. By thinking about a bank this way, we found a new rationale for demandable debt, a new type of bank run—a "money run"—and a new explanation for the other quintessential things banks do, such as pooling assets and maturity/liquidity transformation. In contrast to the literature, our results suggest that financial fragility may be a necessary evil and that regulating markets may help bank stability just as much as regulating banks themselves.

A Proofs

A.1 Proof of Proposition 1

For an instrument i, let $\max v_i$ be an instrument's debt capacity, i.e. its maximum value over any R, r, and σ:

$$\max v_i := \sup \left\{ v_i \,\middle|, r \leq \ell\,,\ R \leq y\,, \sigma \in [0,1] \right\}. \tag{33}$$

So, C_0 lends against instrument i only if $\max v_i \geq c$. Hence, B can fund itself with the puttable loan but not with the loan if and only if

$$\max v_{\text{loan}} < c \leq \max v_{\text{putt. loan}}. \tag{34}$$

Substituting $r = \ell$ and $R = y$ into the expressions for their values in equations (2) and (4) gives the condition in the proposition. □

A.2 Proof of Lemma 1

When C_t and H_t are matched, H_t has been hit by a liquidity shock. Thus, C_t's value of the bond is v_t and H_t's value of the bond is zero (since H_t consumes only at Date t and the bond is not demandable). The total surplus is thus v_t, which C_t and H_t split in proportions $1-\eta$ and η, in accordance with the Nash bargaining solution. Thus the price is $p_t = \eta v_t$. □

A.3 Proof of Proposition 2

The proof is analogous to that of Proposition 1. B can borrow via a bond but not with a puttable loan if and only if

$$\max v_{\text{putt. loan}} < c \leq \max v_{\text{bond}}, \tag{35}$$

where $\max v$ is as defined in equation (33). Substituting $r = \ell$, $R = y$, $\sigma = 1$ into the expressions for their values in equations (4) and (7) gives the condition in the proposition. □

A.4 Proof of Lemma 2

When C_t and H_t are matched H_t has been hit by a liquidity shock. Thus, C_t's value of the banknote is v_t and H_t's value of the banknote is r (since H_t consumes only at Date t, it redeems on demand if it does not trade). The gains from trade are thus $v_t - r$, which C_t and

H_t split in proportions $1-\eta$ and η, in accordance with the Nash bargaining solution, i.e. p_t is such that

$$\text{H}_t \text{ gets} \quad \eta\big(v_t - r\big) + r = p_t, \tag{36}$$

$$\text{C}_t \text{ gets} \quad (1-\eta)\big(v_t - r\big) = v_t - p_t, \tag{37}$$

or $p_t = \eta v_t + (1-\eta)r$. □

A.5 Proof of Proposition 3

The proof is analogous to those of Proposition 1 and Proposition 2. B can borrow via a banknote but not with a puttable loan or a bond if and only if

$$\max\Big\{ \max v_{\text{putt. loan}}\,,\, \max v_{\text{bond}} \Big\} < c \leq \max v_{\text{b.note}}, \tag{38}$$

where $\max v$ is as defined in equation (33). Substituting $r = \ell$, $R = y$, $\sigma = 1$ into the expressions for their values in equations (4), (7), and (10) gives the condition in the proposition. □

A.6 Proof of Proposition 4

For the pure equilibria, the argument is in the text (see equations (13) and (14)).

For the mixed equilibrium, C_t must be indifferent between entering and staying out, $k = v - p$, or

$$k = \frac{\rho(1-\eta)\big(R-r\big)}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{39}$$

Solving for σ gives the expression in the proposition. □

A.7 Proof of Corollary 1

The result follows immediately from Proposition 4. □

A.8 Proof of Corollary 2

We prove points (i)–(iii) in turn.

(i) Consider C_t's best response given other counterparties play σ. C_t enters if

$$k \leq v - p = \frac{\rho(1-\eta)\big(R-r\big)}{\rho + (1-\rho)\theta(1-\eta\sigma)}. \tag{40}$$

The RHS above is decreasing in r (for fixed σ).

(ii) The result follows immediately from equation (13).

(iii) The result follows immediately from equation (14). $\square$

A.9 Proof of Lemma 3

We prove the result by setting up B's maximization problem over R and r given $\sigma = f(R-r)$ and showing that B optimally sets $r = \ell$. We proceed in the following steps.

(i) We write down B's utility as a function of R and r.

(ii) We set up the constrained maximization problem to find R and r.

(iii) We show that the constraint in the maximization problem binds.

(iv) We show that the objective in the maximization problem is increasing in r given the constraint binds.

(v) We conclude that $r = \ell$, its maximum possible value.

B's utility. Let u denote B's expected utility, which can be written recursively as

$$u_t = \rho(y-R) + (1-\rho)\Big(\theta\big(\sigma_t u_{t+1} + (1-\sigma_t)(\ell - r)\big) + (1-\theta)u_{t+1}\Big) \tag{41}$$

The terms are determined as follows. With probability ρ, B's investment pays off and B repays R, keeping $y - R$. With probability $(1-\rho)\theta$, B's investment does not payoff and the debtholder H_t is hit by a liquidity shock. With conditional probability σ_t, H_t finds a counterparty and B continues its investment, getting u_{t+1}, since there is no discounting. Otherwise, with conditional probability $1-\sigma_t$, H_t does not find a counterparty and redeems on demand. B must liquidate its investment and repay r, so it gets $\ell - r$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. Again, B continues and gets u_{t+1}. Given ($u_t = u_{t+1} \equiv u$), substituting $\sigma_t \equiv f \equiv f(R-r)$ in accordance with the hypothesis of the proposition and solving for u gives

$$u = \frac{\rho(y-R) + (1-\rho)\theta(1-f)(\ell - r)}{\rho + (1-\rho)\theta(1-f)}. \tag{42}$$

B's maximization problem. B will choose R and r to maximize u subject to the constraint that $v \geq c$ (so C_0 lends). Substituting for u form equation (42) and for v from equation (10) with $\sigma = f(R-r)$, this reads:

$$\begin{cases} \text{maximize} & \dfrac{\rho\big(y-R\big)+(1-\rho)\theta(1-f)(\ell-r)}{\rho+(1-\rho)\theta(1-f)} \\ \text{s.t.} & \dfrac{\rho R+(1-\rho)\theta(1-\eta f)r}{\rho+(1-\rho)\theta(1-\eta f)} \geq c. \end{cases} \tag{43}$$

Constraint binds. To show that the constraint binds, we show that decreasing R (i) increases the objective and (ii) tightens the constraint:

(i) By differentiation, $\frac{\partial u}{\partial R} < 0$ as long as

$$f'\Big[(1-\rho)\theta\big(y-R-(\ell-r)\big)\Big] < \rho+(1-\rho)\theta(1-f) \tag{44}$$

If the term in square brackets is negative, this is always satisfied, since $f' > 0$. If it is positive, then it is satisfied as long as f' is sufficiently small, which is required by hypothesis.

(ii) By differentiation, $\frac{\partial v}{\partial R} > 0$ as long as

$$f' > -\frac{\rho+(1-\rho)\theta(1-\eta f)}{(1-\rho)\theta\eta(R-r)}. \tag{45}$$

This is always satisfied given $f' > 0$.

Optimal r. To show that $r = \ell$, we show that u is increasing in r given the constraint binds. To see this, compute the total derivative of $u = u\big(r, f(R-r), R(r)\big)$ "along the constraint":

$$\frac{du}{dr} = \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f}\frac{df}{dr} + \frac{\partial u}{\partial R}\frac{dR}{dr} \tag{46}$$

$$= \frac{\partial u}{\partial r} + \frac{\partial u}{\partial f}f'\left(\frac{dR}{dr}-1\right) + \frac{\partial u}{\partial R}\frac{dR}{dr}, \tag{47}$$

where dR/dr comes from differentiating the constraint (given it binds) and the partial derivatives follow from direct computation:

$$\frac{dR}{dr} = -\frac{\theta(1-\rho)\Big((1-\eta f)(\rho+\theta(1-\rho)(1-\eta f)) - \eta\rho(R-r)f'\Big)}{\rho\left(\rho+\theta(1-\rho)(1-\eta f)+\eta\theta(1-\rho)(R-r)f'\right)}, \tag{48}$$

$$\frac{\partial u}{\partial R} = -\frac{\rho}{\rho+(1-\rho)\theta(1-f)}, \tag{49}$$

$$\frac{\partial u}{\partial f} = \frac{(y-R-(\ell-r))\theta(1-\rho)\rho}{(\rho+(1-\rho)\theta(1-f))^2}, \tag{50}$$

$$\frac{\partial u}{\partial r} = -\frac{(1-\rho)\theta(1-f)}{\rho+(1-\rho)\theta(1-f)}. \tag{51}$$

Substituting equations (48), (49), (50), and (51) into equation (47) and manipulating, we see that the derivative $du/dr > 0$ as long as so long as

$$\begin{aligned}\Big[\big(\rho+\theta(1-\rho)(1-\eta f)\big)^2\big(y-\ell-(R-r)\big)+\eta\big(\rho+(1-\rho)\theta(1-f)\big)^2\big(R-r\big)\Big]f' < \\ < (1-\eta)f\big(\rho+(1-\rho)\theta(1-\eta f)\big)\big(\rho+(1-\rho)\theta(1-f)\big).\end{aligned} \tag{52}$$

If the term in square brackets is negative, this is always satisfied, since $f' > 0$. If it is positive, then it is satisfied as long as f' is small, which is required by hypothesis. □

A.10 Proof of Proposition 5

We first solve for the values v^0 and v^1 in terms of r and R given the strategies $\sigma^0 = 0$ and $\sigma^1 = 1$. We then show that these strategies are indeed best responses (for some k). Finally, we argue that $r = \ell$ and compute the repayment R. Finally, we substitute r and R back into the values to get the expressions in the proposition. Then, the fact that B can borrow via the banknote and only via the banknote for λ sufficiently small follows immediately from Proposition 3 and the continuity of v^1 in λ.

Values. From equation (18) with $\sigma^0 = 0$, we have immediately that

$$v^0 = \frac{\rho R+(1-\rho)\theta r}{\rho+(1-\rho)\theta} \tag{53}$$

(this is just the value of the puttable loan in equation (4)). From Lemma 2 (the logic of which is not affected by the presence of sunspots), we have the prices

$$p^0 = \eta v_0 + (1-\eta)r, \tag{54}$$

$$p^1 = \eta\left(\lambda v^0 + (1-\lambda)v^1\right) + (1-\eta)r. \tag{55}$$

Thus, equation (19) with $\sigma^1 = 1$ reads

$$v^1 = \rho R + (1-\rho)\left(\theta\left(\eta\left(\lambda v^0 + (1-\lambda)v^1\right) + (1-\eta)r\right) + (1-\theta)\Big(\lambda v^0 + (1-\lambda)v^1\Big)\right), \quad (56)$$

so

$$v^1 = \frac{\rho R + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta)\big)v_0 + \theta(1-\eta)r\Big)}{\rho + (1-\rho)\Big(\lambda\big(1-\theta(1-\eta)\big) + \theta(1-\eta)\Big)}. \quad (57)$$

Best responses. $\sigma^1 = 1$ and $\sigma^0 = 0$ are best responses if

$$v^0 - p^0 \ \le\ k\ \le\ v^1 - p^1 \quad (58)$$

or

$$v^0 - \eta v^0 - (1-\eta)r\ \le\ k\ \le\ v^1 - \eta\left(\lambda v^0 + (1-\lambda)v^1\right) - (1-\eta)r. \quad (59)$$

This is satisfied for some k as long as $v^1 \ge v^0$, which is the case as long as $R \ge r$, which must be the case since $R > c > \ell \ge r$.

Repayments. $r = \ell$ since v^1 is (uniformly) increasing in r but, for λ small, B's payoff does not depend on r (directly).[32,33]

Now, the repayment R is determined by solving

$$c = \lambda v^0 + (1-\lambda)v^1. \quad (60)$$

Substituting in for v^0 and v^1 from equations (53) and (57) and solving for R, we find

$$R = c + \frac{(1-\rho)\theta\Big(\rho\big(\lambda + (1-\lambda)(1-\eta)\big) + (1-\rho)\big(\lambda + (1-\lambda)\theta(1-\eta)\big)\Big)}{\rho\Big(\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)\Big)}\Big(c - \ell\Big), \quad (61)$$

as expressed in the proposition.

[32]Intuitively, if you are "close" to the good equilibrium (so the banknote almost always circulates), you get all of the benefit increasing r (via the increased price), but almost none of the cost (via the increased payout given early redemption). Formally, $\frac{\partial v}{\partial r} > 0$ uniformly in λ, but $\frac{\partial u}{\partial r} \to 0$ as $\lambda \to 0$ (see the expressions for B's payoffs in equations (67) and (68)).

[33]Note that we are calculating the optimal values of R and r as if they do not affect the equilibria of the $t > 0$ subgames. I.e. counterparties enter in state $s_t = 1$ and not in state $s_t = 0$, as described in the proposition, off the equilibrium path as well as on it. However, other equilibria are possible too, supported by different off-equilibrium behavior.

We can then use the expressions for R and v^0 above and substitute them into v_1 to find

$$v^1 = \frac{\Big(\rho + (1-\rho)\big(\lambda(1+\theta\eta) + (1-\lambda)\theta\big)\Big)c - (1-\rho)\lambda\theta\eta\ell}{\rho + (1-\rho)\big(\lambda + (1-\lambda)\theta\big)}, \tag{62}$$

as expressed in the proposition. □

A.11 Proof of Corollary 3

The results follow directly from differentiation given the expression for R in equation (22). □

A.12 Proof of Proposition 6

We first solve for B's Date-0 utility if it issues a bond, which we label $u|_{\text{bond}}$. Then we solve for B's utility if it issues a banknote, which we label $u|_{\text{b.note}}$. Then we show $u|_{\text{b.note}} \geq u|_{\text{bond}}$ whenever $\lambda \leq \lambda^*$.

Bond. Suppose B issues a bond. By assumption, the bond always circulates. Hence, B never liquidates early and eventually gets y and repays R. Since there is no discounting, B's utility is $u = y - R$. Since the bond is like a banknote that always circulates with redemption value zero, R is given by equation (22) with $\lambda = 0$ and ℓ replaced by zero (since $r = 0$ instead of $r = \ell$). We have

$$u = y - \frac{\rho + (1-\rho)\theta(1-\eta)}{\rho}c =: u\,\Big|_{\text{bond}}. \tag{63}$$

Banknote. Suppose B issues a banknote. Denote B's utility in state s_t by u^{s_t}. First, consider $s_t = 0$. u^0 solves

$$u^0 = \rho(y - R) + (1-\rho)\big(\theta(\ell - r) + (1-\theta)u^0\big), \tag{64}$$

where the terms are determined as follows. With probability ρ, B's investment pays off and B repays R, keeping $y - R$. With probability $(1-\rho)\theta$, B's investment does not payoff and the debtholder H_t is hit by a liquidity shock. Since $s_t = 0$, $\sigma_t = 0$ and H_t redeems on demand and B must liquidate its investment and repay r, getting $r - \ell$. With probability $(1-\rho)(1-\theta)$, B's investment does not pay off and H_t is not hit by a liquidity shock. B gets u^0, since $s_{t+1} = 0$ given $\mathbb{P}\left[s_{t+1} = 0 \,|\, s_t = 0\right] = 1$. Solving for u^0 with $r = \ell$ gives

$$u^0 = \frac{\rho(y - R)}{\rho + (1-\rho)\theta}. \tag{65}$$

Now, consider $s_t = 1$. u^1 solves

$$u^1 = \rho(y - R) + (1 - \rho)\big(\lambda u^0 + (1 - \lambda)u^1\big), \tag{66}$$

where the terms are determined as follows. With probability ρ, B's investment pays off and B repays R, keeping $y - R$. With probability $1 - \rho$, B's investment does not payoff. In this case, B continues its investment to the next date (it does not matter if H_t is shocked, since B's debt always circulates given $s_t = 1$). Hence, with conditional probability λ, $s^{t+1} = 0$ and B gets u^0 and, with conditional probability $1 - \lambda$, $s^{t+1} = 1$ and B gets u^1. Solving for u^1 gives

$$u^1 = \frac{\rho(y - R) + (1 - \rho)\lambda u^0}{\rho + (1 - \rho)\lambda}. \tag{67}$$

B's Date-0 utility is thus

$$u_{\text{b.note}} = \lambda u^0 + (1 - \lambda)u^1. \tag{68}$$

Substituting for u^0, u^1, and R from equations (65), (67), and (22) and differentiating immediately gives the following lemma, which is useful below.

Lemma 4. *$u\,\big|_{\text{b.note}}$ is continuously decreasing in λ.*

Proof. Direct computation gives

$$\frac{\partial}{\partial\lambda}\Big(u\,\big|_{\text{b.note}}\Big) = -\frac{(1-\rho)\theta\Big(\rho y - \big(\rho + (1-\rho)\theta(1-\eta)\big)c + \rho\eta(c-\ell) + (1-\rho)\theta(1-\eta)\ell\Big)}{\big(\rho + (1-\rho)\theta\big)\big(\lambda + (1-\lambda)\rho\big)^2}. \tag{69}$$

This is negative since each term in the numerator is positive, given $\rho y - (\rho + (1-\rho)\theta(1-\eta))c \geq 0$ by the assumption that the bond is feasible (equation (8)).

□

Comparison. B prefers to issue a banknote than a bond whenever $u|_{\text{b.note}} \geq u|_{\text{bond}}$. From the expressions above, equality holds if

$$\lambda^* = \frac{\rho\big(\rho + (1-\rho)\theta\big)(1-\eta)\ell}{\rho(y-\ell) + \big(\rho + (1-\rho)\theta\big)(1-\eta)(\rho\ell - c)}. \tag{70}$$

And given $u|_{\text{bond}}$ does not depend on λ and $u|_{\text{b.note}}$ is increasing in λ (Lemma 4), $u|_{\text{b.note}} \geq u|_{\text{bond}}$ exactly when $\lambda \leq \lambda^*$. □

A.13 Proof of Proposition 7

Now a banknote's redemption value r is bounded not by the liquidation value ℓ of a single investment, but by the liquidation $N\ell$ of the pool of investments. Still, it is bounded by C_t's entry condition: $r \leq r^{\max}$ (equation (28)). This is the tighter bound, since $r^{\max} \leq N\ell$ by the assumption in equation (25). Hence, we modify the definition of $\max v$ (equation (33)) to be

$$\max v := \sup \left\{ v \,\middle|\, r \leq r^{\max}\,,\ R \leq y\,, \sigma \in [0,1] \right\}. \tag{71}$$

Substituting $r = r^{\max}$, $R = y$, and $\sigma = 1$ into the value of the banknote (equation (10)) gives

$$\max v = y - \frac{(1-\rho)\theta}{\rho}\, k. \tag{72}$$

Requiring that $\max v \geq c$, i.e. that Date-0 creditors lend, gives the condition of the proposition. □

A.14 Proof of Proposition 8

By Proposition 3, B can invest in (y', ℓ') but not in (y, ℓ) if and only if

$$\max v_{\text{b.note}}\big|_{(y,\ell)} < c \leq \max v_{\text{b.note}}\big|_{(y',\ell')}, \tag{73}$$

where $\max v$ is as defined in equation (33). Substituting for R, r and σ in the value of the banknote (equation (10)), this says that

$$\frac{\rho y + (1-\rho)\theta(1-\eta)\ell}{\rho + (1-\rho)\theta(1-\eta)} < c \leq \frac{\rho y' + (1-\rho)\theta(1-\eta)\ell'}{\rho + (1-\rho)\theta(1-\eta)}. \tag{74}$$

There exists c satisfying the above inequalities whenever the left-most term is less than the right-most term. This reduces to the condition in the proposition (equation (29)). □

A.15 Proof of Proposition 9

Observe first that the value of the banknote is given by the same expression as in the baseline model (equation (10)). But now an interior value of σ is determined by counterparties' entry condition. Recall that the matching function is homogenous, so each counterparty is matched with a debtholder with probability σ/q. Counterparties' entry condition is thus

$$\frac{\sigma}{q}\big(v - p\big) \geq k, \tag{75}$$

where q represents the steady-state mass of counterparties entering at each date. Since each counterparty is small, the inequality above will bind. Substituting in for v and $p = \eta v + (1-\eta)r$, we have

$$\frac{\sigma}{q}\left(\frac{\rho(1-\eta)\big(R-r\big)}{\rho+(1-\rho)\theta(1-\eta\sigma)}\right) = k. \tag{76}$$

With $\sigma = \mu\sqrt{q}$, this can be re-written as

$$\mu k(1-\rho)\theta\eta q - k\big(\rho+(1-\rho)\theta\big)\sqrt{q} + \mu\rho(1-\eta)(R-r) = 0. \tag{77}$$

This is a quadratic equation in $\sqrt{q}$. It has the two solutions, i.e. there are two steady states,

$$\sqrt{q}_{\pm} = \frac{k\big(\rho+(1-\rho)\theta\big) \pm \sqrt{k^2\big(\rho+(1-\rho)\theta\big)^2 - 4\mu^2 k\rho(1-\rho)(R-r)\theta\eta(1-\eta)}}{2\mu k(1-\rho)\theta\eta}. \tag{78}$$

Substituting $\sigma_{\pm} = \mu\sqrt{q_{\pm}}$ gives the expressions in the proposition. □

A.16 Proof of Proposition 10

Given $\tilde{k}_t \sim \mathrm{U}\left[0,\bar{k}\right]$, we can replace σ in C_t's entry condition (equation (40)) by $\mathbb{P}\left[\sigma_t = 1\right] = k^*/\bar{k}$, . C_t must be indifferent at the cut-off k^*:

$$k^* = \frac{\rho(1-\eta)\big(R-r\big)}{\rho+(1-\rho)\theta\big(1-\eta k^*/\bar{k}\big)}. \tag{79}$$

Rearranging gives a quadratic equation which has the two solutions in equation (32), as long as they are well defined, i.e. $k^{\pm} \in [0,1]$, as required by the proposition. □

B Table of Notations

	Players and Indices
t	time index
B	borrower or "bank"
C_t	(potential) creditor/counterparty at Date t
H_t	debtholder at Date t
	Technologies and Preferences
y	payoff of B's investment
c	cost of B's investment
ℓ	liquidation value of B's investment
ρ	probability B's investment pays off each date
θ	probability C_t is hit by liquidity shock at each date
u	B's utility (used only in the Appendix)
	Prices, Values, and Strategies
R	terminal repayment (face value of debt)
r	redemption value
v_t	value of B's debt to a creditor at Date t
p_t	secondary-market price of B's debt at Date t
σ_t	mixed strategy of counterparty C_t
	Other Variables
s_t	sunspot at Date t (Subsection 4.3)
λ	$\mathbb{P}\left[s_{t+1}=0 \mid s_t=1\right]$, "confidence crisis" probability (Subsection 4.3)
$\max v$	debt capacity/maximum value of an instrument (equation (33))
$r^{\max}$	maximum redemption value s.t. C_t enters (equation (28))
μ	matching parameter in Subsection 5.2

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