

# Conflicting Priorities: A Theory of Covenants and Collateral

JASON RODERICK DONALDSON, DENIS GROMB, and GIORGIA PIACENTINO\*

## ABSTRACT

We develop a theory of secured debt, unsecured debt, and debt with anti-dilution covenants. We assume that, as in practice, covenants convey the right to accelerate if violated, but the new secured debt retains its priority even if issued in violation of covenants. We find that such covenants are nonetheless useful: They provide state-contingent financing flexibility, balancing over- and underinvestment incentives. The optimal debt structure is multilayered, combining secured and unsecured debt with and without covenants. Our results are consistent with observations about debt structure, covenant violations, and waivers. They speak to a policy debate about debt priority.

FIRMS FINANCE THEMSELVES MAINLY WITH debt, often combining different types of debt.<sup>1</sup> Some types of debt have stronger priority rights than others.

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Correspondence: Giorgia Piacentino, USC Marshall School of Business, 3670 Trousdale Parkway, Los Angeles, CA 90089; e-mail: [gpiacent@marshall.usc.edu](mailto:gpiacent@marshall.usc.edu).

<sup>1</sup> See, for example, Erel et al. (2012) on debt's predominance (95.6% in their sample) and, for example, Barclay and Smith (1995), Rauh and Sufi (2010), and Colla, Ippolito, and Li (2013) on its heterogeneity.

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Notably, secured debt has absolute priority over assets used as collateral.<sup>2</sup> Creditors are therefore vulnerable to dilution by future secured debt issuance. One defense against dilution is secured debt itself: Its absolute priority prevents new debt from taking priority. Another defense is so-called negative pledge covenants, which, if violated by a secured debt issuance, convey the right to accelerate payment.<sup>3</sup>

Such anti-dilution covenants are no substitute for collateral, however, as new secured debt retains its priority even if issued in violation of covenants. Pointing to this weakness, lawyers advise against relying on them.<sup>4</sup> Indeed, some, such as Bjerre (1999), seem puzzled by their very existence:

The [negative pledge] covenant does not prevent third parties from acquiring a security interest, but [is] merely...a hollow promise, for in the very act of breaching the covenant, the borrower places its assets out of reach of the negative pledgee and into the hands of the very third party against which the negative pledgee seeks protection. (p. 308)

Yet, many firms employ unsecured debt and negative pledge covenants even when collateral is available.<sup>5</sup> Why? What determines the mix of secured and unsecured debt with and without negative pledge covenants?

To address these questions, we develop a model in which collateral serves to establish priority among different debt claims, overriding any covenants in place. The power of property rights (collateral) to override contractual rights (covenants) is central to our theory. Yet, despite being a good approximation of the law, it is all but absent from the existing finance literature, the lone exception being Ayotte and Bolton (2011).

In the model, a borrower chooses his debt structure to manage a trade-off between financial commitment and flexibility. By issuing secured debt, he commits not to dilute it in the future, which can prevent overinvestment. But without the flexibility to dilute, he might be unable to take on new debt, which can cause underinvestment. We show how debt structures that combine secured and unsecured debt with and without covenants can be used to manage the trade-off between over- and underinvestment—they can block “bad dilution” but allow “good dilution.” Our results are consistent with several observations about debt structure, including covenant violations and waivers. They also speak to the debate about the efficiency of current priority rules.

<sup>2</sup> See, for example, Hansmann and Kraakman (2002) and Merrill and Smith (2001) on the priority of secured debt over unsecured debt and equity. In practice, unsecured debt is sometimes paid ahead of secured debt, but only in 11% of Chapter 11 bankruptcies and never in Chapter 7 bankruptcies (Bris, Welch, and Zhu (2006)).

<sup>3</sup> In Billett, King, and Mauer's (2007) sample, negative pledge covenants are the fourth most common covenants, included in 44% of debt contracts.

<sup>4</sup> For example, an article in the *National Law Review* states that “a Negative Pledge is merely an unsecured promise and gives the Lender very little” (“Negative Pledge Pros and Cons,” April 10, 2016), a view that is widely held among lawyers (see, for example, D'Angelo and Saccomandi (2016) and Goetz and Hoffmann (2010)).

<sup>5</sup> See, for example, Badoer, Dudley, and James (2019), Benmelech, Kumar, and Rajan (2019), Lian and Ma (2019), and Rampini and Viswanathan (2013).

*Model preview.* A borrower, B, has two risky projects. Project 0's value is positive; Project 1's value, revealed after Project 0 is underway, can be positive or negative. Once underway, projects can be liquidated at a private cost to B.

B must raise funds from creditors. He issues debt to finance Project 0 and, later, possibly more debt to finance Project 1. Financing is subject to two frictions. First, pledgeability is limited in that projects' values comprise not only cash flow, which can be pledged to creditors, but also private benefits, which cannot. Second, contracts are nonexclusive in that contracting with one creditor cannot prevent contracting with another.

These frictions lead to distinct roles for covenants and collateral. Negative pledge covenants are promises not to take on new secured debt. If the promise is broken by a secured debt issuance, the debt contracts are in conflict—upholding one violates the other. We assume that collateral resolves such conflicts by establishing priority: Debt secured by collateral trumps other debt, including debt protected by covenants. Covenant-protected debt has one defense against such dilution, namely, the right to accelerate, but even in this case the new secured debt retains its priority.

*Results preview.* Our first set of results, in Propositions 1 and 2, characterizes the conditions under which B invests efficiently in Project 1 if he finances Project 0 via only unsecured and/or secured debt. A tension exists between over- and underinvestment. On the one hand, unsecured debt can lead to overinvestment when Project 1's value is negative. Indeed, due to nonexclusivity, B can finance Project 1 with secured debt, and thereby transfer its cost onto existing unsecured creditors while capturing its private benefits. Thus, dilution by new secured debt can be bad. On the other hand, financing Project 0 via secured debt alone can lead to underinvestment in Project 1 when its value is positive. The reason is that, due to limited pledgeability, Project 1's cost may be less than the debt capacity it creates (its expected cash flow) even if its value (private benefits included) is positive. As a result, diluting existing debt could be necessary to finance the project. In this case, dilution can be good, and preventing such dilution via secured debt can lead to underinvestment, or a "collateral-overhang problem" as in Donaldson, Gromb, and Piacentino (2020a). We show that a suitable mix of unsecured and secured debt can foster efficient investment if and only if the underinvestment problem is "mild" relative to the overinvestment problem, in the sense that positive-value projects have a smaller financing shortfall than negative-value projects. If so, then, secured debt can indeed be set at a level that reduces B's borrowing capacity sufficiently to prevent investment if Project 1's value is negative while leaving it sufficient to finance Project 1 if its value is positive.

Our next result, in Proposition 3, characterizes the conditions under which B invests efficiently in Project 1 if he finances Project 0 via only unsecured debt, with and without negative pledge covenants. A priori, covenants can deter dilution via the threat of acceleration and the costly liquidation it may imply. However, we find a negative result: If all debt is covenant-protected, the acceleration threat is not credible—covenants have no teeth (subject to a condition on outstanding debt). Acceleration is unattractive as it not only fails to



undo dilution by new secured debt, but can even subsidize it—by forcing risky projects to be liquidated, it reduces their risk, which benefits secured debt.

We next show, in Proposition 4, that with less covenant-protected debt and more unprotected debt, the acceleration threat can be credible. The reason is that acceleration allows protected debt to get paid first in time, which is valuable insofar as one can leapfrog unprotected debt and grab something. As a result, it turns out that a suitable mix of covenant-protected and unprotected debt can foster efficient investment whenever unsecured and secured debt cannot, that is, when the underinvestment problem is “severe,” in that positive-value projects have a larger financing shortfall than negative-value projects. In this case, the covenant-protected creditor has more to gain from acceleration after a negative-value project, and so upholds covenants after a negative-value projects and waives them otherwise. In sum, we show that anti-dilution covenants and collateral each have a role to play despite covenants’ weakness.

Finally, in Proposition 5, we characterize the equilibrium debt structure, which involves secured and unsecured debt with and without covenants. Covenants are sometimes violated, and when they are, they are waived—the acceleration is “off path.” This result rationalizes why covenants exist despite being rarely enforced and suggests that collateral and covenants are complementary tools in managing agency problems. The debt structure is consistent with several stylized facts, including that well-capitalized firms, which are unlikely to dilute existing debt, rely heavily on unsecured debt, whereas other firms use a mix of debt instruments, use negative pledge covenants, violate covenants, receive covenant waivers from creditors following violations, use covenants and collateral as parts of multitiered debt structures, and have less financial flexibility when they have more secured debt.

The results also speak to the policy debate around the strong priority of secured debt (Bebchuk and Fried (1996)).<sup>6</sup> In particular, the results suggest a counterargument to the concern that it harms unsecured debt, as borrowers in our model choose a debt structure to commit not to engage in bad dilution, while maintaining the flexibility to engage in good dilution. (Creditors, meanwhile, are compensated fairly for dilution risk.)

*Literature.* We add to the large finance theory literature on debt structure.<sup>7</sup> In most of that literature, covenants and collateral mitigate conflicts between borrowers and creditors; in our paper, they mitigate conflicts among creditors, as in DeMarzo (2019), Donaldson, Gromb, and Piacentino (2020a,

<sup>6</sup> Other legal studies on secured debt and priority include Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984, 1994, 1997); papers on contracting subject to legal rules include Aghion and Hermalin (1990), Gennaioli (2006), and Ravid et al. (2015).

<sup>7</sup> For more on collateral, see, for example, Bester (1985), Eisfeldt and Rampini (2009), Hart and Moore (1994, 1998), and Rampini and Viswanathan (2010, 2013). For more on covenants, see, for example, Berlin and Mester (1992), Gârleanu and Zwiebel (2009), Park (2002), and Rajan and Winton (1995). There are also numerous other papers on debt structure without covenants, for example, Bolton and Scharfstein (1996), Gennaioli and Rossi (2013), and Gertner and Scharfstein (1991).



2020b), Longhofer and Santos (2003), and Stulz and Johnson (1985). Collateral mitigates conflicts between creditors and derivatives holders in Bolton and Oehmke (2015) and between creditors and asset buyers in Donaldson, Gromb, and Piacentino (2021). In Attar et al. (2019b), covenants not only mitigate conflicts among creditors, but facilitate collusion among them.<sup>8</sup>

The spirit of our paper, in which “hard claims” are chosen to constrain investment but could constrain it too much, follows Hart and Moore (1995), in which debt and equity are chosen to control an empire-building manager. We show how different claims—covenants and collateral—emerge as complementary ways to manage the over-/underinvestment trade-off. We show that, unlike those in Hart and Moore (1995), these claims are effective even when they can be freely renegotiated.

Closest to our paper, Ayotte and Bolton (2011) also study negative pledge covenants. Like our paper, they analyze how property and priority rights affect investment, and they rationalize aspects of current law. We go a step further by considering efficient dilution, rationalizing covenant violations and waivers, and studying acceleration and renegotiation.

*Layout.* Section I presents the model. Section II reports the results, and Section III considers extensions. Section IV discusses related evidence, new predictions, and other implications. Section V concludes. All proofs are in Appendix A. In Appendix B, we show our findings are robust to other modeling assumptions.

## I. Model

We develop a three-date model,  $t \in \{0, 1, 2\}$ , of a borrower B who funds two projects sequentially. There is universal risk neutrality, limited liability, and no discounting.

### A. Projects

Borrower B is penniless but has two projects: Project 0 at Date 0 and Project 1 at Date 1. Project 0 costs  $I_0$  at Date 0 and pays off at Date 2: With probability  $p$ , it succeeds and yields cash flow  $X_0 > 0$  and private benefit  $Y_0 > 0$ ; otherwise, it fails and pays nothing. Its value is positive:

$$p(X_0 + Y_0) > I_0. \quad (1)$$

Project 1 costs  $I_1$  at Date 1 and pays off at Date 2: With probability  $p$ , it succeeds and yields cash flow  $X_1^Q > 0$  and private benefit  $Y_1^Q > 0$ , which depend on its quality  $Q \in \{H, L\}$ ; otherwise, it fails and pays nothing. Quality  $Q$  is revealed at Date 1, with  $\mathbb{P}[Q = H] =: q$ . The project has positive value if and

<sup>8</sup> Other papers on nonexclusive financial contracting include Acharya and Bisin (2014), Asryan and Vanasco (2024), Attar et al. (2019a), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and DeMarzo (1992), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).

only if its quality is high:

$$p(X_1^H + Y_1^H) > I_1 > p(X_1^L + Y_1^L). \quad (2)$$

We assume for simplicity that the two projects are perfectly correlated, that is, both succeed or both fail; for example, Project 1 could be an extension of Project 0.<sup>9,10</sup>

Projects can be liquidated at Date 1 for their expected cash flow  $p(\mathbb{1}_0 X_0 + \mathbb{1}_1 X_1^Q)$ , with  $\mathbb{1}_t := 1$  if Project  $t$  is undertaken, and  $\mathbb{1}_t := 0$  otherwise. This is what competitive buyers would bid for the projects if, as we assume below, only B enjoys  $Y_t$ . (We abstract from partial and random liquidation for simplicity only.)

### B. Financing

*Frictions.* At Date  $t \in \{0, 1\}$ , B can raise  $I_t$  from competitive creditors under two frictions.

1. *Limited pledgeability:* B cannot transfer more than the projects' cash flow to creditors, that is, he cannot pledge private benefits.<sup>11</sup>
2. *Nonexclusive contracting:* B's contract with initial creditors at Date 0 cannot prevent or constrain new contracts with other creditors at Date 1.

Nonexclusivity does not mean that B's contract with initial creditors cannot contain a covenant not to enter new contracts with other creditors, but rather that such covenants can always be violated. Although punishments for such violations could establish effective exclusivity, they are limited by limited pledgeability. In that sense, nonexclusivity can be seen as deriving from limited pledgeability.<sup>12</sup>

*Instruments.* We focus on three financing instruments.

1. *Secured debt* is a promise to pay a fixed face value at Date 2 with projects as collateral. (The role of collateral is determined by the priority rules described below.)
2. *Unsecured debt* is a promise to pay a fixed face value at Date 2, without collateral.

<sup>9</sup> This assumption simplifies the analysis by reducing the number of outcomes (see, however, Appendix B).

<sup>10</sup> Under this interpretation,  $I_1$  can also represent a "liquidity shock" that must be paid to preserve the value of a single project.

<sup>11</sup> Private benefits can be linked to control (Aghion and Bolton (1992), Grossman and Hart (1988)) and can also capture output that B diverts or opportunities created by the project but out of the legal reach of creditors. Private benefits can also be mapped explicitly to cash diversion models: With  $X_1^L = 0$ , undertaking the negative-value project can be interpreted as diverting  $I_1$  to get  $Y_1^L$  at deadweight cost  $I_1 - Y_1^L$ .

<sup>12</sup> Other foundations for nonexclusivity are possible. One is imperfect information. That is not suitable here, as we assume that all contracts, and covenant violations in particular, are observable.

3. *Unsecured debt with negative pledge covenants* (hereinafter “covenant-protected debt”) is like unsecured debt but also conveys the right to accelerate if B takes on new secured debt, that is, to demand payment at Date 1 of part or all of the face value.<sup>13</sup> Covenants can be waived both ex post and ex ante, that is, creditors can opt not to accelerate after a violation or to give up their right to in anticipation of one.

We show that focusing on these claims is without loss of generality in the sense that no agent can benefit from other claims being introduced. We also assume that a single creditor holds all unsecured debt with negative pledge covenants but no other type of debt. This turns out to be optimal ex ante, and hence also without loss of generality, and allows us to sidestep creditor coordination issues (e.g., runs and hold-outs).<sup>14</sup>

*Priority rules.* Given nonexclusivity, B can enter into conflicting contracts with different creditors, that is, can violate covenants. The following priority rules resolve such conflicts.

1. *Secured debt has priority over unsecured debt:* Unsecured debt is not paid unless secured debt is paid in full.
2. *Secured debt issued earlier has priority over that issued later:* Secured debt taken at Date 1 is not paid unless that taken at Date 0 is paid in full.
3. *Unsecured debt (with or without covenants) due earlier has priority over that due later:* Unsecured debt due at Date 2 is not paid unless that due at Date 1 (possibly due to acceleration) is paid in full.

We assume that in the event that not all claims are satisfied at a given date—a so-called “payment default”—all debt is accelerated and paid according to priority. In this case, all unsecured debt becomes due at the same time and, thus, is *pari passu*.

These rules reflect practice.<sup>15</sup> Moreover, together with the available instruments, they are (weakly) optimal in our model (see Proposition 5).

### C. Timeline and Solution Concept

To summarize, the sequence of moves is as follows. At Date 0, B chooses whether and how to fund Project 0 from competitive creditors.<sup>16</sup> At Date 1,

<sup>13</sup> In Appendix B, we study the case in which creditors can demand payment of only none or all of the face value.

<sup>14</sup> This captures the case of a lender holding a large stake or of multiple lenders coordinating to waive covenants, for example, via collective action clauses, secondary market trading, or active cooperation. Indeed, even for public firms’ bonds, covenants can be and are often modified by simple majority vote (Kahan and Tuckman (1993), Bratton (2006)). More generally, coordination among dispersed bondholders does not seem to delay restructuring (Helwege (1999)).

<sup>15</sup> See, for example, Hahn (2010), Merrill and Smith (2001), and Schwartz (1989).

<sup>16</sup> We assume that if B funds Project 0, he borrows exactly  $I_0$ , which we show to be (weakly) optimal (Proposition 5).



sequentially, (i) the quality  $Q$  is revealed; (ii) B funds Project 1, raising  $I_1$  via secured debt from competitive creditors or not;<sup>17</sup> (iii) if a covenant is violated, covenant-protected creditors choose a fraction of their debt to accelerate; and (iv) debt, if due, is paid in full or projects are liquidated and debt is paid according to priority. At Date 2, (i) projects that have not been liquidated succeed or fail (together) and (ii) debt, if due, is paid according to priority. Additionally, contracts can be renegotiated if B and all of his creditors can be made strictly better off at any point time. We assume, for simplicity, that B has the bargaining power in such renegotiations. (Section III.A discusses renegotiation by subcoalitions of parties.)

We solve for the subgame perfect equilibrium.

#### D. Assumptions

We now impose two restrictions on the parameters.

**ASSUMPTION 1:** *Under efficient project choice, expected cash flows exceed funding needs, that is if B undertakes Project 0 and subsequently undertakes Project 1 only if it is efficient to do so (i.e.,  $Q = H$ ), then expected cash flows exceed funding needs:*

$$pX_0 - I_0 + q(pX_1^H - I_1) \geq 0. \quad (3)$$

This assumption ensures that limited pledgeability alone does not prevent B from implementing the efficient investment policy. Hence, nonexclusivity is necessary for our results.

**ASSUMPTION 2:** *Irrespective of Project 1's quality, the expected cash flow of both projects exceeds the face value needed to finance Project 1, that is, for  $Q \in \{H, L\}$ ,*

$$p(X_0 + X_1^Q) > \frac{I_1}{p}. \quad (4)$$

Thus, the projects' liquidation value suffices to repay the secured debt needed to finance Project 1.

#### E. First-Best Policy

The first-best policy is to undertake a project if and only if it has positive value and never to liquidate (which destroys private benefits).

**LEMMA 1:** *The first-best policy is to undertake Project 0, to undertake Project 1 if and only if  $Q = H$ , and not to liquidate at Date 1.*

<sup>17</sup> As we show in Section III.A, this merely imposes restrictions on B's behavior that he would otherwise choose to adopt, and can impose with realistic instruments that we abstract from in the baseline model.

## II. Results

We study whether the first-best can obtain via a suitable debt structure at Date 0. Limited pledgeability and nonexclusivity both present potential impediments. These frictions create a tension between the need to pursue good dilution to loosen financial constraints stemming from limited pledgeability and the temptation to pursue bad dilution made possible by nonexclusivity.

Although we have to analyze each case to solve for the equilibrium, the tension can be illustrated in general. As Project 1's value includes not only cash flow, which is pledgeable, but also private benefits, which are not, funding could require pledging some unencumbered cash flow from Project 0, possibly expropriating value from existing creditors. That could be good if it boosts new debt capacity to fund investment if  $Q = H$  (good dilution) but bad if it boosts the temptation to invest even if  $Q = L$  (bad dilution).

### A. Unsecured Debt

We first study when the first-best can be implemented by borrowing solely unsecured at Date 0.

Say B funded Project 0 at Date 0 with unsecured debt with face value  $F_0^u$ . At Date 1, B can pledge all cash flows—from Project 0 and Project 1—to new secured creditors, possibly diluting existing unsecured debt. B's debt capacity is thus  $p(X_0 + X_1^Q)$  and funding Project 1 is feasible provided it covers  $I_1$ , which it does by Assumption 2. Underinvestment is therefore not a concern: Unsecured debt at Date 0, which can be diluted at Date 1, ensures that B has enough debt capacity to invest if  $Q = H$ , that is it allows good dilution.<sup>18</sup>

By the same token, however, unsecured debt may allow bad dilution, leading to possible overinvestment. The following proposition characterizes when the first-best nonetheless obtains.

PROPOSITION 1 (UNSECURED DEBT): *Define*

$$Y_1^* := \min \left\{ X_0 - \frac{I_0}{p}, X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left( X_0 - \frac{I_0}{p} + X_1^H - \frac{I_1}{p} \right) \right\}. \quad (5)$$

*The first-best is implementable via unsecured debt without covenants at Date 0 if and only if*

$$Y_1^L \leq Y_1^* \quad \text{or} \quad p(X_0 + qX_1^H + (1-q)X_1^L) \geq I_0 + I_1. \quad (6)$$

The idea is as follows. The first inequality says that private benefit  $Y_1^L$  is too small to tempt B into an investment with negative net present value (NPV). If

<sup>18</sup> In Stulz and Johnson (1985), diluting unsecured debt with new secured debt relaxes a debt-overhang problem (Myers (1977)), thus improving investment efficiency. Here, dilution instead loosens financial constraints. Moreover, renegotiation in our context does not remove the need for dilution whereas it would in Stulz and Johnson (1985).

instead  $Y_1^L > Y_1^*$ , B is tempted to overinvest but his existing unsecured creditors will renegotiate their debt to “bribe” him not to. The second inequality says that, despite the anticipated bribe, B’s creditors are still willing to lend at Date 0. If neither holds, unsecured debt financing is infeasible as paying the bribe when  $Q = L$  at Date 1 is so costly for creditors that funding Project 0 is infeasible at Date 0. We focus on this case from here forward.

**ASSUMPTION 3:** *Unsecured debt cannot implement the first-best: Condition (6) is violated.*

### B. Secured Debt

We now ask when the first-best can be implemented by borrowing via a mix of secured and unsecured debt at Date 0.

**PROPOSITION 2 (SECURED AND UNSECURED DEBT):** *The first-best is implementable via a mix of secured and unsecured debt without negative pledge covenants at Date 0 if and only if*

$$X_1^L \leq X_1^H. \quad (7)$$

The idea is as follows. At Date 1, B can promise new secured creditors all cash flows not already promised to secured debt at Date 0. This dilutes existing unsecured debt, but not existing secured debt. Date 1 financing is thus feasible as long as

$$p(X_0 + X_1^Q - F_0^s) \geq I_1, \quad (8)$$

where  $F_0^s$  is the face value of the secured debt taken at Date 0.

The first-best is implemented if and only if financing is feasible when it is efficient, and not when it is not, or, put differently, when good dilution is possible but bad dilution is not (i.e., when inequality (8) holds if  $Q = H$  but not if  $Q = L$ ). That can happen for some amount of secured debt  $F_0^s$  when the left-hand side (LHS) of the inequality is larger for a high-quality project than a low-quality one or, per condition (7), when  $X_1^H \geq X_1^L$ .

That condition can be interpreted as the underinvestment problem being relatively “mild,” in the sense that the financing shortfall is smaller for positive-value projects than negative-value projects. Otherwise, the underinvestment problem is relatively “severe,” and secured and unsecured debt cannot implement the first-best. Indeed, any level of secured debt preventing investment when  $Q = L$  would do so too when  $Q = H$ . (This is the “collateral-overhang problem” in Donaldson, Gromb, and Piacentino (2020a).)

### C. Negative Pledge Covenants

We now ask under what conditions the first-best can be implemented by borrowing at Date 0 via a mix of unsecured debt with and without negative pledge



covenants. If B issues new secured debt at Date 1, the covenant-protected creditor can accelerate. Although that does not undo dilution, as the new secured debt retains its priority, it forces liquidation, which destroys B's private benefits.<sup>19</sup> The acceleration threat therefore has the potential to deter (over-) investment. We find, however, that too much covenant protection can be self-defeating.

**PROPOSITION 3 (COVENANT IRRELEVANCE):** *Suppose B financed Project 0 at Date 0 via unsecured debt with face value  $F_0^u$ , a fraction  $\phi$  of which is covenant-protected, and that  $Q = L$  at Date 1.*

- *If  $X_0 + X_1^L \geq F_0^u + I_1/p$ , B will not finance Project 1 irrespective of whether this would trigger acceleration, that is, the outcome is the same as for  $\phi = 0$ .*
- *Otherwise, financing Project 1 triggers acceleration if and only if  $\phi \leq \phi^*$  with*

$$\phi^* := 1 - \frac{(1-p)I_1/p}{p(X_0 + X_1^L - I_1/p)} \in (0, 1). \quad (9)$$

The idea is as follows. The first part of the propositions says that there is no dilution if  $X_0 + X_1^L \geq F_0^u + F_1^s$ , where  $F_1^s = I_1/p$  is the face value of (secured) debt used to finance Project 1. In that case, covenants have no bite, as B would bear the costs of investment and therefore have no incentive to invest in a negative-value project anyway. The second part of the proposition says that if there is dilution, covenants have bite only if the proportion of covenant-protected debt  $\phi$  is small enough. To see why, consider the cost and benefit of acceleration in this case:

- The cost is the subsidization of secured debt. Indeed, acceleration forces liquidation, leading secured debt  $F_1^s$ , which retains its priority, to be paid with probability one rather than  $p$ . The secured debt's gain is thus  $(1-p)F_1^s$ .<sup>20</sup>
- The benefit is the dilution of the unprotected unsecured debt. Absent acceleration, unprotected unsecured debt shares cash flows net of secured debt payment with the covenant-protected debt, so its payoff is  $(1-\phi)p(X_0 + X_1^L - F_1^s)$ . Under acceleration, it would get nothing since the

<sup>19</sup> This is reminiscent of the disciplining role of demand deposits in the banking literature (e.g., Calomiris and Kahn (1991) and Diamond and Rajan (2001)) and of short-term debt more broadly (e.g., Bolton and Scharfstein (1990)). One distinction is that here covenant-protected debt can be accelerated only in the event of a covenant violation. Moreover, the threat is credible only if some debt is not covenant-protected (per Proposition 3).

<sup>20</sup> This expression makes use of Assumption 2, which implies that liquidation proceeds suffice to repay secured debt in full, or  $p(X_0 + X_1^Q) > F_1^s$ . If the assumption were violated, secured debt would get everything after liquidation and accelerated debt would get nothing, rendering the acceleration threat noncredible. In that case, covenants would again be irrelevant. See Section III.A for a discussion of how other instruments could have a role to play in such cases.

other debts are paid all cash flows at Date 1, so acceleration implies a loss to unprotected unsecured debt of  $(1 - \phi)p(X_0 + X_1^L - F_1^s)$ .<sup>21</sup>

Since liquidation leaves the expected cash flows unchanged, it is a zero-sum game among creditors. Thus, acceleration benefits the covenant-protected debt if the unprotected unsecured debt's loss exceeds the secured debt's gain, which amounts to  $\phi \leq \phi^*$ .<sup>22</sup>

Note that for  $\phi = 1$ , acceleration has no benefit. For a smaller fraction  $\phi$ , however, the benefit could outweigh the cost, making the threat credible. We therefore ask whether a  $\phi$  exists that can implement the first-best.

**PROPOSITION 4 (COVENANTS):** *For the first-best to be implementable via a mix of unsecured debt with and without negative pledge covenants at Date 0, a sufficient condition is*

$$X_1^L \geq X_1^H. \quad (10)$$

*This condition is also necessary if, additionally,  $p(X_0 + X_1^H) - I_1 < \bar{F}_0^u$ , with  $\bar{F}_0^u$  defined as the unique solution to  $I_0 = qz + (1 - q)p \min\{X_0, z\}$ .*

The idea is as follows. Unless debt is renegotiated, implementing the first-best requires that the threat be credible if  $Q = L$  but not if  $Q = H$ . In this case, B will not violate the covenant to invest in negative-value projects, knowing it will be upheld, but does invest in positive-value projects, knowing it will be waived.<sup>23</sup> As the cost of accelerating is  $(1 - p)F_1^s$  no matter  $Q$ , the benefit  $(1 - \phi)p(X_0 + X_1^Q)$  must be greater for  $Q = L$  than  $Q = H$ . That is ensured by the condition  $X_1^L \geq X_1^H$ , which, intuitively, states that there is more to grab if  $Q = L$  than if  $Q = H$ , so the covenant-protected creditor has a stronger incentive to accelerate and get it ahead of other unsecured creditors. This corresponds to the underinvestment problem being “severe,” in the sense that positive-value projects have a relatively larger financing shortfall than negative-value projects.

We note that B may also implement the first-best with renegotiation when  $Q = H$ , in effect B “bribing” the creditors into not accelerating. The condition

<sup>21</sup> This assumes that the covenant-protected creditor accelerates exactly  $p(X_0 + X_1^Q) - F_1^s$ , which can be repaid from the liquidation proceeds, rather than accelerating all of its debt, which would trigger bankruptcy and in turn accelerate other unsecured debt. (Appendix B considers an extension in which acceleration necessarily triggers bankruptcy.)

<sup>22</sup> That acceleration is most attractive for low  $\phi$  contrasts with the result in Gennaioli and Rossi (2013) that liquidation bias is strongest when a controlling creditor's share is high. The difference comes from the fact that in their paper the controlling creditor is senior/secured and hence has the most to gain from liquidation.

<sup>23</sup> Here, B violates the covenant, which the creditor then waives. This is equivalent to B asking the creditor to waive the covenant before he takes secured debt, that is, there is no distinction between asking for “forgiveness” and “permission.” This suggests that covenant violations could be more frequent than measures of ex post violations imply, especially since, in practice, asking for “permission” could allow a borrower to circumvent any direct costs of covenant violation, beyond the risk of acceleration that we model (e.g., due to lost reputation). We thank Adriano Rampini for pointing this out.

in the proposition that  $p(X_0 + X_1^H) - I_1$  be small enough ensures that B does not have sufficient financial slack to do so (analogous to Proposition 1).

#### D. Optimal Debt Structure

Our analysis implies that a debt structure always exists that implements the first-best. Which instruments are effective depends on the projects. Collateral is optimal if positive-value projects have higher cash flows than negative-value projects ( $X_1^H \geq X_1^L$ );<sup>24</sup> otherwise covenants are optimal.

**PROPOSITION 5 (DEBT STRUCTURE):** *The equilibrium policy is first-best efficient. There exist values  $F_0$ ,  $F_0^s$ , and  $\phi$  such that the policy can be implemented as follows.*

*At Date 0, B finances Project 0 by borrowing  $I_0$  with the following debt structure.*

- *If Assumption 3 is violated, the debt is unsecured without covenants with face value  $F_0$ .*
- *Otherwise,*
  - *if  $X_1^H > X_1^L$ , the debt is a mix of secured and unsecured debt without covenants with face values  $F_0^s$  and  $F_0 - F_0^s$ ;*
  - *if  $X_1^H \leq X_1^L$ , the debt is a mix of unsecured debt with and without covenants, with face values  $\phi F_0$  and  $(1 - \phi)F_0$ .*

*At Date 1:*

- *If  $Q = H$ , negative pledge covenants, if any, are waived and B finances Project 1 by borrowing  $I_1$  via secured debt with face value  $I_1/p$ .*
- *If  $Q = L$ , B does not finance Project 1.*

The optimal debt structure can be understood as follows. Limited pledgeability reduces debt capacity, which sometimes leads to underinvestment when  $Q = H$ . If it does not, B can rely on unsecured debt without covenants. If it does, B must overcome the underinvestment problem via a multilayered debt structure that depends on the problem's severity: If the problem is mild, B mixes secured and unsecured debt; if it is severe, he mixes unsecured debt with and without covenants. In either case, he invests in Project 1 when  $Q = H$ , with the project financed by secured debt that dilutes debt in place. Covenants therefore implement efficiency only in concert with collateral. Although they are needed to promise not to use collateral when dilution is inefficient, collateral is needed to break that promise and engage in good dilution.

This result suggests a role for observed priority rules: They allow debt to be structured so as to implement an efficient investment policy by permitting

<sup>24</sup> Collateral may not be necessary for optimality, however; an appropriate mix of covenant-protected and unprotected debt is also optimal if the necessary condition in Proposition 4 does not hold.



borrowers to dilute unsecured debt when, but only when, needed to overcome financial constraints that would otherwise impede efficient investment.

### III. Extensions

Here we discuss several twists of the model, mainly to explain other type of debt instruments (Section III.A) but also to show robustness to more than just binary qualities (Section III.B).

#### A. Other Instruments

Our baseline model considers secured debt and unsecured debt with and without covenants. These three debt claims suffice to achieve the first-best. However, variations of the model can suggest a rationale for other types of debt that arise in practice.

1. *Secured debt with covenants* arises if we relax the assumption that B raises exactly  $I_1$  at Date 1 (see note<sup>17</sup>). It allows B to commit not to borrow more than  $I_1$  at Date 1, per the baseline, which is indeed optimal (Proposition 5). To see why, suppose B issues secured debt at Date 0 with a clause prohibiting new debt in excess of  $I_1/p$ , backed by an option to accelerate if borrowing at Date 1 exceeds this “deductible.” The threat is credible as the covenant-protected secured creditors are paid for sure in the event of acceleration (per the argument in Section II.C). Thus, B prefers not to violate the covenant to avoid acceleration and the liquidation it implies.
2. *Unsecured debt with covenants limiting unsecured debt* arises if we relax the assumption that any Date 1 borrowing must be secured (see note<sup>17</sup>). It allows B to commit not to borrow unsecured at Date 1, per the baseline, which is indeed optimal (Proposition 5). To see why, suppose B finances Project 0 with such covenant-protected unsecured debt. If B takes on unsecured debt at Date 1, then the covenant-protected creditor will accelerate its debt and be paid in full—in that sense, acceleration does undo dilution by unsecured debt—so acceleration at Date 1 is always attractive and B prefers to issue secured debt at Date 1.
3. *Interc Creditor agreements* (among different types of debt in the same issuance) arise if we relax the assumption that renegotiation must make all parties better off, that is, if we allow B to renegotiate with some creditors at the expense of others.<sup>25</sup> Such renegotiation could undermine the acceleration threat, since B could offer the covenant-protected creditor collateral as a bribe not to accelerate, allowing it to be paid ahead of other unsecured debt without forcing liquidation.<sup>26</sup> Hence, B would like credi-

<sup>25</sup> We thank Ken Ayotte for pointing this out.

<sup>26</sup> This would be a second lien, paid after  $F_1^s$  but ahead of  $F_0^u$ . Its payoff from accepting the bribe would thus be (up to)  $p(X_0 + X_1^Q - F_1^s)$ , which exceeds his payoff from acceleration of  $p(X_0 + X_1^Q) - F_1^s$ .

tors to commit not to engage in such renegotiation (Proposition 5). To do so, B can include an intercreditor agreement in its Date 0 issuance specifying that one creditor cannot change its claim without consent of the others. This deters new debt at Date 1.

4. *Subordinated debt*, with priority below other unsecured debt, arises if we relax the assumption that there are only two qualities. It allows B to finance projects of “very high quality” without violating covenants, while preserving the baseline outcome for high- and low-quality projects, which is optimal (Proposition 5). To see why, assume an additional project quality  $Q = HH$ , which is self-financing, that is,  $pX_1^{HH} > I_1$ . Such a project could be financed by issuing subordinated debt without harming existing debt. However, due to the dilution benefits of acceleration (Section II.C), the issuance could still lead to acceleration if it violates a covenant, which could deter efficient investment. A covenant prohibiting all but subordinated debt could therefore be preferable.

### B. Continuum of Qualities

Above we show that secured (resp. covenant-protected) debt is optimal when the underinvestment problem is relatively mild (severe), that is, when positive-value projects are more (less) pledgeable than negative-value projects. Here we show that, under some simplifying assumptions, these results are robust to including more than two qualities under a condition on the distribution of qualities. We then show that when the condition is violated, not even a mix of secured and covenant-protected debt can implement the first-best.

Suppose Project 1 has a continuum of possible qualities  $Q$ , with the value of a project with quality  $Q$  given by  $p(X_1^Q + Y_1^Q) - I_1$ , which is assumed to be increasing in  $Q$ . For simplicity, we assume that (i) there is no renegotiation and (ii) covenant-protected debt is never paid in full after being diluted with secured debt. From Section II.B, we know that for a given level of secured debt  $F_0^s$ , B can fund Project 1 if

$$X_1^Q \geq \frac{I_1}{p} - (X_0 - F_0^s) =: X_1^{Q^s}, \quad (11)$$

and from Section II.C, we know that for a given fraction  $\phi$  of unsecured debt with covenants, B can fund Project 1 if

$$X_1^Q < \frac{1 - p\phi}{p(1 - \phi)} F_1^s - X_0 =: X_1^{Q^c}, \quad (12)$$

where  $F_1^s = I_1/p$  from Date 1 creditors' break-even condition.

Equations (11) and (12) say that B can commit to invest only in projects with  $X_1^Q \geq X_1^{Q^s}$  via secured debt and only in projects with  $X_1^Q \leq X_1^{Q^c}$  via covenants. So if every positive-value project has higher  $X_1^Q$  than any negative-value project, then the first-best is implemented via secured debt with  $F_0^s$  chosen such that  $p(X_1^{Q^s} + Y_1^{Q^s}) = I_1$ ; likewise, if every positive-value project has lower

$X_1^Q$  than any negative-value project, then the first-best is implemented via a fraction  $\phi$  of covenant-protected debt chosen such that  $p(X_1^{Q^c} + Y_1^{Q^c}) = I_1$ .<sup>27</sup> Our results therefore generalize when there is a clear relationship between the sign of projects' value and their cash flow/pledgeability.<sup>28</sup>

We next show that if there is no such relationship, then not only does the first-best not obtain with secured or covenant-protected debt, but it does not obtain with a mix either. In this case, there must be either two positive-value projects with qualities  $H_0$  and  $H_1$  and one negative-value project with value  $L$  such that  $X_1^{H_0} < X_1^L < X_1^{H_1}$  or two negative-value projects and one positive-value project satisfying an analogous condition. As the analysis is the same in these two cases, we focus on the first. With an amount  $F_0^s$  of secured debt and a fraction  $\phi$  of unsecured covenant-protected debt in place, B invests, per the analysis above, if  $X_1^Q \in [X_1^{Q^s}, X_1^{Q^c}]$  (where the definitions of  $X_1^{Q^s}$  and  $X_1^{Q^c}$  are as in equations (11) and (12) except  $F_1^s$  is replaced by the total amount of secured debt  $F_0^s + F_1^s$  in the latter). Thus, for B to invest in positive- but not negative-value projects—that is, for the first-best to be implemented—it must be the case that  $X_1^{H_0}, X_1^{H_1} \in [X_1^{Q^s}, X_1^{Q^c}]$ , but  $X_1^L \notin [X_1^{Q^s}, X_1^{Q^c}]$ , which contradicts the assumption that  $X_1^L \in (X_1^{H_0}, X_1^{H_1})$ . Thus, when neither secured nor covenant-protected debt implements the first-best, a mix will not do so either.

#### IV. Empirical Content and Discussion

In this section, we describe the empirical relevance of our results, as well as their practical and theoretical implications.

##### A. Empirical Relevance

Our model is consistent with a number of stylized facts about corporate debt structure: (i) per Proposition 1, well-capitalized/highly rated firms, corresponding to those with high cash flows or minimal incentive problems (Assumption 3), use unsecured debt (e.g., Rauh and Sufi (2010) and Benmelech, Kumar, and Rajan (2024)); (ii) per Proposition 4, negative pledge covenants are common (Billett, King, and Mauer (2007) and Ivashina and Vallée (2018)) but, per Proposition 5, covenants are frequently violated and/or renegotiated/waived

<sup>27</sup> A sufficient condition for every positive-value project to have higher (lower)  $X_1^Q$  than any negative-value projects is that  $X_1^Q$  be increasing (decreasing) in  $Q$ .

<sup>28</sup> That is not to say that covenants and collateral cannot add value when those conditions are violated, as they can still deter negative-value projects. Here, absent covenants or collateral, B would always pursue Project 1, making the ex interim surplus  $E[X_1^Q + Y_1^Q - I_1]$ . With, for example, covenants, this surplus would be  $E[1_{\{X_1^Q \leq X_1^{Q^c}\}}(X_1^Q + Y_1^Q - I_1)]$ , making the efficiency gain from optimal covenants the maximum difference:  $\max_{X_1^{Q^c}} E[(1 - 1_{\{X_1^Q \leq X_1^{Q^c}\}})(X_1^Q + Y_1^Q - I_1)]$ . Keep in mind, however, that renegotiation, which is ruled out here, can also achieve such ex interim efficiency improvements. A full analysis of the value of covenants requires studying how covenants and collateral create ex ante financing capacity in the baseline environment with renegotiation (see Assumption 3 and the preceding discussion).



(e.g., Beneish and Press (1993, 1995), Chava and Roberts (2008), Dichev and Skinner (2002), Gopalakrishnan and Prakash (1995), Nini, Smith, and Sufi (2012), Roberts and Sufi (2009), and Sweeney (1994));<sup>29</sup> and (iii) per Proposition 4, covenants in some debt decrease the yield on other debt because they decrease default risk (e.g., Bradley and Roberts (2015)).<sup>30</sup>

We note that papers often study covenant violations in general and do not single out negative pledge covenants. As some covenants could be based on financial ratios such as interest to earnings, not all violations need be intentional. Some could result only from, for example, low earnings. See Bjerre (1999) for case law on violations of negative pledge covenants specifically.

### B. New Predictions

In our theory, an optimal debt structure arises from the trade-off between the costs of financial flexibility (allowing overinvestment) and its benefits (avoiding underinvestment). Firms are exposed to underinvestment problems if  $X_1^H$  is low, so financing high-quality projects is hard, and overinvestment problems if  $X_1^L$  is low, so financing low-quality projects is easy.<sup>31</sup> This leads to the following predictions, which to the best of our knowledge have yet to be tested directly, but appear consistent with existing indirect evidence (see Proposition 5).

**PREDICTION 1:** All else equal, firms that are relatively more exposed to underinvestment problems use covenants.

Proxies for underinvestment problems could be growth opportunities, financial constraint indices, high fixed costs, inflexible investment needs, strong rivals (so underinvestment leads to decreased market share), and nonredeployable assets (so underinvestment leads to dormant assets).

**PREDICTION 2:** All else equal, firms that are relatively more exposed to overinvestment problems use collateral.

Dilution problems are likely to be severe in firms in distress and in declining industries, as they have an incentive to gamble for resurrection, tunnel, strip assets, and shift risk.

To the extent that what creditors can seize in the model ( $X_0$  and  $X_1^Q$ ) reflects asset tangibility, we have the following prediction.

<sup>29</sup> Upholding negative pledge covenants stays off path all of the time in the model and, most of the time in practice as well. There are examples, however, of their being successfully upheld, consistent with their providing a credible threat in the model; see, for example, McDaniel (1983).

<sup>30</sup> See Green (2018), Greenwald (2019), and Matvos (2013) for structural models on the value of covenants.

<sup>31</sup> This mapping from cash flows in the model to ease of financing in practice is predicated on incentive distortions stemming from debt-equity conflicts and from the debt dilution option in particular. These problems are likely to be of first order for high-credit-risk firms. For other firms, managerial agency problems could be first order. The latter firms, however, typically use neither covenants nor collateral, but instead rely on other devices to manage incentives (see, for example, Jensen (1986)).

**PREDICTION 3:** Collateral use increases and covenant use decreases with the tangibility of assets used in (good) investment opportunities.

Tangible assets include real estate, equipment, and inventories (see, for example, Rampini and Viswanathan (2013)).

**PREDICTION 4:** Covenant use decreases with the costs associated with asset sales.

Asset sale discounts (see Appendix B) are likely to be higher when assets are less redeployable, less tangible, harder to value, or more firm-specific and when potential buyers are scarce, outside the industry, or financially constrained.

## V. Conclusion

In this paper, we present a model in which financial contracts are nonexclusive: Although contracts may include covenants that put restrictions on other contracts, the covenants can be violated. Financial contracts can therefore be in conflict, in which case priority rules are needed to resolve such conflicts.

In practice, secured debt enjoys strong priority. This allows dilution of unsecured debt by new secured debt, increasing the cost of debt and reducing debt capacity. Negative pledge covenants, which convey the right to accelerate following new secured debt issuance, have limited bite—they generally cannot reverse the issuance of secured debt or the priority it enjoys over collateral. Thus, the wide use of negative pledge covenants—contracts that rule out secured debt but that are themselves defeated by it—is puzzling. For instance, Bjerre (1999), argues that

Some may wonder why, given their weakness, costs, and difficulties, lenders bother with negative pledge covenants at all.... [B]orrowers have strong incentives to breach the covenant if necessary financing is available only on a secured basis. [...] The foregoing simply raises, however, the broader question of why lenders ever agree to lend on an unsecured basis, with or without a negative pledge covenant, if collateral is available. (pp. 338–339)

We find that, on the one hand, dilution by secured debt can be good: It can prevent underinvestment by loosening borrowing constraints due to limited pledgeability. On the other hand, when dilution can lead to over-investment, it can be bad. Secured debt, whose priority rights prevent dilution, can block it. Dilution can also be blocked by a mix of unsecured debt with and without negative pledge covenants, as the acceleration threat discourages new secured debt issuance. The acceleration threat is credible not because it reverses the issuance of secured debt, but rather because it allows accelerated debt to dilute other unsecured debt.

In sum, covenants can implement efficiency only in concert with collateral. Although covenants are needed to commit not to use collateral when dilution

is inefficient, collateral is needed to break that promise and engage in “good dilution.”

More broadly, our analysis contributes to the policy debate about the strong priority rights of secured debt. The results suggest that such priority rules can play a useful role by allowing debt dilution when it is efficient, but safeguarding against it when it is not.

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## Appendix A: Proofs

### Proof of Proposition 1

Say B can only issue unsecured debt at Date 0. An equilibrium is efficient if Project 0 is funded, and Project 1 is funded provided  $Q = H$ . Because creditors are competitive, B gets all of the surplus. Hence, if any equilibrium is efficient, all are. We derive conditions for an efficient equilibrium to exist.

By Assumption 2, B is able to fund Project 1 at Date 1, borrowing secured debt with face value  $I_1/p$ . By condition (2), B is also willing to do so if  $Q = H$ ,

$$p(Y_1^H + \max\{0, X_0 + X_1^H - I_1/p - F_0^u\}) \geq p \max\{0, X_0 - F_0^u\}. \quad (A1)$$

Since Project 1 is efficient if  $Q = H$ , this is renegotiation-proof. Thus, two types of efficient equilibria may exist a priori, which we call Type 1 and Type 2. In Type 1 equilibria, B will not fund Project 1 if  $Q = L$ , which is an efficient, and thus renegotiation-proof, outcome. In Type 2 equilibria, B would fund Project 1 if  $Q = L$ , but this inefficient outcome is renegotiated away.

We consider each type of equilibria in turn after a preliminary lemma.

**LEMMA A.1:** *An efficient equilibrium exists if and only if B's Date 0 unsecured debt capacity exceeds  $I_0$ .*

**PROOF:** We show that if B fund Project 0 via unsecured debt, B takes the efficient action at Date 1 for both  $Q = H$  and  $Q = L$ . If  $Q = H$ , B invests per equation (A1). Now suppose  $Q = L$ . Since investing is inefficient, renegotiation would see creditors bribe B not to invest whenever he has the incentive and ability to do so.  $\square$

*Type 1 equilibria:* Since creditors are competitive, if, in a Type 1 equilibrium, B borrows with unsecured debt with face value  $F_0^u$ , B raises an amount equal to creditors' expected payoff under the efficient Date 1 policy. This amount is a function  $f$  of the face value of debt:

$$f(z) := p(q \min\{z, X_0 + X_1^H - I_1/p\} + (1 - q) \min\{z, X_0\}). \quad (A2)$$

**LEMMA A.2:** *A unique  $\hat{F}_0^u$  exists such that  $f(\hat{F}_0^u) = I_0$ .*

**PROOF:** Let  $F_0^{\max} := \max\{X_0 + X_1^H - I_1/p, X_0\}$ .  $f$  is continuous and strictly increasing over  $[0, F_0^{\max}]$  and constant over  $[F_0^{\max}, \infty)$ , with  $f(0) = 0$  and



$f(F_0^{\max}) = p(q(X_0 + X_1^H - I_1/p) + (1-q)X_0) \geq I_0$ , by Assumption 1. Hence, by the Intermediate Value Theorem,  $\hat{F}_0^u$  exists and is unique.  $\square$

A Type 1 equilibrium exists, by definition, only if B will not fund Project 1 when  $Q = L$ , or

$$p(Y_1^L + \max\{0, X_0 + X_1^L - I_1/p - \hat{F}_0^u\}) \leq p \max\{0, X_0 - \hat{F}_0^u\}, \quad (\text{A3})$$

where  $\hat{F}_0^u$  is as defined in Lemma A.2.

LEMMA A.3: Condition (A3) is equivalent to  $Y_1^L \leq X_0 - \hat{F}_0^u$ .

PROOF: Condition (A3) can be rewritten as

$$Y_1^L \leq \max\{0, X_0 - \hat{F}_0^u\} - \max\{0, X_0 - \hat{F}_0^u + X_1^L - I_1/p\}, \quad (\text{A4})$$

which, because condition (2) implies  $X_1^L - I_1/p < 0$ , can be rewritten as

$$Y_1^L \leq \max\{0, \min\{X_0 - \hat{F}_0^u, I_1/p - X_1^L\}\}. \quad (\text{A5})$$

In turn, because  $Y_1^L \geq 0$ , this is equivalent to

$$Y_1^L \leq \min\{X_0 - \hat{F}_0^u, I_1/p - X_1^L\},$$

which, because  $Y_1^L \leq I_1/p - X_1^L$  (by condition (2)), amounts to  $Y_1^L \leq X_0 - \hat{F}_0^u$ .  $\square$

LEMMA A.4:  $Y_1^L \leq X_0 - \hat{F}_0^u$  is equivalent to  $Y_1^L \leq X_0 - \tilde{F}_0^u$ , with  $\hat{F}_0^u$  as defined in Lemma A.2 and  $\tilde{F}_0^u$  the unique solution to

$$I_0 = p(q \min\{X_0 + X_1^H - I_1/p, z\} + (1-q)z) =: g(z). \quad (\text{A6})$$

PROOF:  $g$  is continuous and strictly increasing with  $g(0) = 0$  and  $\lim_{y \rightarrow \infty} g(y) = \infty$ . So, by the Intermediate Value Theorem,  $\tilde{F}_0^u$  exists and is unique.

If  $Y_1^L \leq X_0 - \hat{F}_0^u$ , then  $\hat{F}_0^u \leq X_0$  and so  $\hat{F}_0^u = \tilde{F}_0^u = \min\{\hat{F}_0^u, X_0\}$ . Conversely, if  $Y_1^L \leq X_0 - \tilde{F}_0^u$ , then  $\tilde{F}_0^u \leq X_0$  and so  $\hat{F}_0^u = \tilde{F}_0^u = \min\{\hat{F}_0^u, X_0\}$ .  $\square$

LEMMA A.5:  $Y_1^L \leq X_0 - \tilde{F}_0^u$  is equivalent to  $Y_1^L \leq Y_1^*$  (with  $\tilde{F}_0^u$  and  $Y_1^*$  as defined in Lemma A.4 and equation (5)).

PROOF: If  $X_0 + X_1^H - I_1/p \geq I_0/p$ , then  $\tilde{F}_0^u = I_0/p$ . Otherwise,  $\min\{\tilde{F}_0^u, X_0 + X_1^H - I_1/p\} = X_0 + X_1^H - I_1/p$ . Hence, in either case,  $\min\{\tilde{F}_0^u, X_0 + X_1^H - I_1/p\} = \min\{I_0/p, X_0 + X_1^H - I_1/p\}$ , which implies

$$\tilde{F}_0^u = \frac{I_0/p - q \cdot \min\{I_0/p, X_0 + X_1^H - I_1/p\}}{(1-q)}. \quad (\text{A7})$$

Hence,  $Y_1^L \leq X_0 - \tilde{F}_0^u$  amounts to  $Y_1^L \leq Y_1^*$ .  $\square$

In sum, if  $Y_1^L \leq Y_1^*$ , B has no incentive to invest in Project 1 when  $Q = L$  and an efficient Type 1 equilibrium exists.

*Type 2 equilibria:* A Type 2 equilibrium exists, by definition, only if B would fund Project 1 when  $Q = L$ . Assume now that  $Y_1^L > Y_1^*$ .

LEMMA A.6: *B's Date 0 unsecured debt capacity is*

$$p(q(X_0 + X_1^H - I_1/p) + (1 - q)(X_0 + X_1^L - I_1/p)). \quad (A8)$$

PROOF: Say that B borrowed with unsecured debt with face value  $F_0^u$ . If  $Q = H$ , then B borrows with secured debt with face value  $I_1/p$  and the unsecured creditors' expected payoff is  $p \min\{X_0 + X_1^H - I_1/p, F_0^u\}$ . If  $Q = L$ , since B has full bargaining power, renegotiation sees creditors get the same expected payoff as in the status quo, that is, if B did invest, which is  $p \cdot \min\{X_0 + X_1^L - I_1/p, F_0^u\}$ . The creditors' Date 0 expected payoff is thus  $p(q \min\{X_0 + X_1^H - I_1/p, F_0^u\} + (1 - q) \cdot \min\{X_0 + X_1^L - I_1/p, F_0^u\})$ , which is (weakly) increasing in  $F_0^u$  and equals the expression in the statement of the lemma for  $F_0^u$  large enough. Being creditors' maximum expected payoff, this expression is also B's Date 0 borrowing capacity.  $\square$

That B's Date 0 borrowing capacity exceeds  $I_0$  follows from condition (6).

### Proof of Proposition 2

LEMMA A.7:  $X_1^H > X_1^L$  is sufficient for an efficient equilibrium to exist.

PROOF: Let  $F_0^s := X_0 + X_1^L - I_1/p + \epsilon$  with  $\epsilon > 0$ , and  $F_0^u = \hat{F}_0^u - F_0^s$ , where  $\hat{F}_0^u$  is as defined in Lemma A.2. For  $\epsilon$  sufficiently small, (i) Assumption 2 implies  $F_0^s \geq 0$  and (ii) Assumption 3 and  $X_1^H > X_1^L$  together imply  $F_0^s < I_0/p$ . We therefore have that  $F_0^u > 0$ .

Now say that B issues secured and unsecured debt with face values  $F_0^s$  and  $F_0^u$ . If so, he raises  $f(F_0^s + F_0^u) \geq f(\hat{F}_0^u) = I_0$  at Date 0 if he follows the efficient Date 1 policy, which we now show he does for  $\epsilon$  small enough. Given  $X_1^H > X_1^L$ , we have

$$X_0 + X_1^H - F_0^s > I_1/p > X_0 + X_1^L - F_0^s. \quad (A9)$$

Thus, B can fund Project 1 via secured debt with face value  $I_1/p$  if  $Q = H$  but not if  $Q = L$ . Condition (2) implies that B is also willing to invest if  $Q = H$ ,

$$p(Y_1^H + \max\{0, X_0 + X_1^H - I_1/p - F_0^s - F_0^u\}) \geq p \max\{0, X_0 - F_0^s - F_0^u\}. \quad (A10)$$

Hence, B invests if  $Q = H$  but not if  $Q = L$ . This outcome is efficient and thus renegotiation-proof.  $\square$

LEMMA A.8: If  $X_1^L \geq X_1^H$ , a necessary condition for an efficient equilibrium to exist in which B issues secured debt with face value  $F_0^s$  is  $I_1 \leq p(X_0 + X_1^H - F_0^s)$ .

PROOF: Say  $I_1 > p(X_0 + X_1^H - F_0^s)$ . Investment requires that both new and existing creditors be better off than in the status quo, and hence be better off

collectively,

$$p \min\{X_0, F_0^u + F_0^s\} \leq p(X_0 + X_1^H) - I_1. \quad (\text{A11})$$

But this cannot hold as (i)  $p(F_0^u + F_0^s) > p(X_0 + X_1^H) - I_1$  by hypothesis and (ii)  $pX_0 > p(X_0 + X_1^H) - I_1$  by the hypothesis that  $X_1^L \geq X_1^H$  and condition (2).  $\square$

LEMMA A.9: *If  $X_1^L \geq X_1^H$ , and if B with borrows secured and unsecured debt with face values  $F_0^s$  and  $F_0^u$  and  $I_1 \leq p(X_0 + X_1^H - F_0^s)$ , then the equilibrium outcome of the Date 1 subgame is the same as if B borrowed with unsecured debt with face value  $(F_0^s + F_0^u)$ .*

PROOF:  $X_1^L \geq X_1^H$  and  $I_1 \leq p(X_0 + X_1^H - F_0^s)$  imply  $I_1 \leq p(X_0 + X_1^L - F_0^s)$ . Thus, B can fund Project 1 if  $Q = H, L$  by borrowing with secured debt. The rest of the proof follows that of Proposition 1 after replacing  $F_0^u$  with  $F_0^s + F_0^u$ .  $\square$

LEMMA A.10:  $X_1^H > X_1^L$  is necessary for an efficient equilibrium to exist.

PROOF: If  $X_1^L \geq X_1^H$ , efficiency requires  $I_1 \leq p(X_0 + X_1^H - F_0^s)$ , which implies that the equilibrium outcome is as if B borrowed with unsecured debt with face value  $(F_0^s + F_0^u)$ , which by Assumption 3 cannot be efficient.  $\square$

#### Proofs of Proposition 3 and Proposition 4

Lemmas A.11 to A.14 characterize the equilibrium of the subgame at Date 1 for  $Q = L, H$ , assuming that B issued unsecured debt at Date 0 with face value  $F_0^u$ , a fraction  $\phi \in (0, 1]$  of which is covenant-protected.

LEMMA A.11: *If B funds Project 1 at Date 1 by borrowing with secured debt with face value  $F_1^s$ , then  $F_1^s \geq I_1$ .*

PROOF: Say  $F_1^s < I_1$ . Absent renegotiation, the Date 1 creditor's payoff is at most  $F_1^s$ . With renegotiation, the creditor's payoff is the same, as B has full bargaining power. But that violates the creditor's break-even constraint, contradicting the hypothesis that it lends.  $\square$

LEMMA A.12: *If  $Q = L$ , B does not borrow with secured debt at Date 1 if doing so would result in acceleration.*

PROOF: If B does not fund Project 1, then his expected payoff is  $Y_0^L + p \max\{X_0 - F_0^u, 0\}$ . If B funds Project 1 with secured debt with face value  $F_1^s$  and acceleration follows, his payoff is  $\max\{p(X_0 + X_1^L) - F_1^s - F_0^u, 0\}$ . Lemma A.11 and condition (2) imply  $pX_1^L < F_1^s$ . Hence,

$$\max\{p(X_0 + X_1^L) - F_1^s - F_0^u, 0\} \leq \max\{pX_0 - F_0^u, 0\} < Y_0^L + p \max\{X_0 - F_0^u, 0\}. \quad (\text{A12})$$

Therefore, B is worse off under acceleration.  $\square$

LEMMA A.13: *If the covenant-protected creditor accelerates some of its debt, it accelerates (only) the maximum amount of debt payable from the liquidation proceeds, that is,  $\min\{p(X_0 + X_1^Q) - F_1^s, \phi F_0^u\}$ .*



PROOF: Define  $\Psi := \min\{p(X_0 + X_1^Q) - F_1^s, \phi F_0^u\}$  and  $\alpha^* := \Psi/(\phi F_0^u) \in [0, 1]$ . Say that the covenant-protected creditor accelerates  $\alpha \phi F_0^u$  with  $\alpha \in [0, 1]$ . For  $\alpha \in [0, \alpha^*]$ , the creditor gets  $\alpha \phi F_0^u$ , which reaches its maximum  $\Psi$  for  $\alpha = \alpha^*$ .

There are two cases corresponding to the two arguments in the “min” in the definition of  $\Psi$ . First, if  $\Psi = \phi F_0^u$ , then  $\alpha^* = 1$  and thus  $\alpha = \alpha^*$  is optimal. Otherwise, if  $\Psi = p(X_0 + X_1^Q) - F_1^s$ , then  $\alpha > \alpha^*$  triggers default, all unsecured debt comes due, and the covenant-protected creditor gets  $\phi \Psi$ . This is less than  $\Psi$ , its payoff for  $\alpha = \alpha^*$ , which is thus optimal.  $\square$

LEMMA A.14: Suppose  $p(X_0 + X_1^Q) \leq F_0^u + I_1$ :

- (i) acceleration is renegotiation proof;
- (ii) funding Project 1 with secured debt with face value  $F_1^s$  triggers acceleration if and only if

$$\phi < \phi^*(X_1^Q; F_0^u, F_1^s) := \frac{X_0 + X_1^Q - F_1^s/p}{\min\{X_0 + X_1^Q - F_1^s, F_0^u\}} > 0; \text{ and} \quad (\text{A13})$$

- (iii)  $\phi^*$  is strictly increasing in  $X_1^Q$  and strictly decreasing in  $F_1^s$ .

PROOF: Statement (i):  $F_1^s \geq I_1$  implies  $p(X_0 + X_1^Q) \leq F_0^u + F_1^s$ . Under acceleration, creditors therefore get the liquidation proceeds,  $p(X_0 + X_1^Q)$ . Absent acceleration, B can only pledge expected cash flows  $p(X_0 + X_1^Q)$ . Hence, creditors cannot all be strictly better off.

Statement (ii): The covenant-protected creditor does not accelerate if and only if

$$\min\{p(X_0 + X_1^Q) - F_1^s, \phi F_0^u\} \leq p\phi \min\{X_0 + X_1^Q - F_1^s, F_0^u\}, \quad (\text{A14})$$

given it accelerates only  $\min\{p(X_0 + X_1^Q) - F_1^s, \phi F_0^u\}$  (Lemma A.13). As the second term under the min on the LHS always exceeds the right-hand side ( $\phi F_0^u > p\phi F_0^u$ ), this inequality can be written as

$$p(X_0 + X_1^Q) - F_1^s \leq p\phi \min\{X_0 + X_1^Q - F_1^s, F_0^u\}. \quad (\text{A15})$$

Solving for  $\phi$  gives the threshold in the lemma.

Statement (iii). The monotonicity of  $\phi^*$  follows from inspection.  $\square$

Lemma A.15 to Lemma A.18 characterize the equilibrium, including B's decision at Date 0.

LEMMA A.15: Let  $\bar{F}_0^u$  be the unique solution to

$$I_0 = qz + (1 - q)p \min\{X_0, z\} =: h(z). \quad (\text{A16})$$

We have that  $I_0 < \bar{F}_0^u < \hat{F}_0^u$  (with  $\hat{F}_0^u$  as defined in Lemma A.2).

PROOF:  $h$  is continuous, strictly increasing with  $h(0) = 0$ , and  $\lim_{z \rightarrow \infty} h(z) = \infty$ . So, by the Intermediate Value Theorem,  $\bar{F}_0^u$  exists and is unique. Moreover,  $h(z) < z$ , so  $\bar{F}_0^u > I_0$ .

We also have  $h > f$  (defined in equation (A2)). Hence,  $h(\hat{F}_0^u) > f(\hat{F}_0^u) = I_0$  by definition of  $\hat{F}_0^u$  (Lemma A.2) and so  $\hat{F}_0^u > \bar{F}_0^u$ .  $\square$

LEMMA A.16: If  $p(X_0 + X_1^H) > \bar{F}_0^u + I_1$ , an efficient equilibrium exists.

PROOF: We construct an efficient equilibrium with  $F_0^u = \bar{F}_0^u$  and  $\phi < \phi^*$ .

Assumption 3 and the hypothesis together with  $\bar{F}_0^u > I_0$  imply that  $p(X_0 + X_1^L) < \bar{F}_0^u + I_1$  and thus  $X_1^H > X_1^L$ . Hence,  $\phi < \phi^*(X_1^Q, F_0^u, F_1^s)$  for  $Q \in \{L, H\}$  and all  $F_1^s \leq I_1/p$  from Lemma A.14 (statement (iii)).

First, consider  $Q = L$ . Raising  $I_1$  requires a face value  $F_1^s \in [I_1, I_1/p]$  and would trigger acceleration as  $\phi < \phi^*$  for  $X_1^Q = X_1^L$  and  $F_1^s \in [I_1, I_1/p]$  (Lemma A.14). Hence, B does not invest (Lemma A.12).

Now consider  $Q = H$  and say that B borrows with face value  $I_1$ . Absent renegotiation, acceleration would follow as  $\phi < \phi^*$  (Lemma A.14). If the liquidation value  $p(X_0 + X_1^H)$  exceeds the total face value  $\bar{F}_0^u + I_1$ , unsecured and secured creditors would get  $\bar{F}_0^u$  and  $I_1$ . However, acceleration is renegotiated away as the expected cash flow  $p(X_0 + X_1^H)$  that B can pledge exceeds creditors' payoff  $\bar{F}_0^u + I_1$ . Since B has full bargaining power, the secured creditor gets payoff  $I_1$  and breaks even, while unsecured creditors get payoff  $\bar{F}_0^u$  and break even as their Date 0 expected payoff is  $q\bar{F}_0^u + (1 - q) \cdot p \min\{X_0, \bar{F}_0^u\} = I_0$ . The Date 1 outcomes are efficient.  $\square$

LEMMA A.17: If  $X_1^L > X_1^H$ , an efficient equilibrium exists.

PROOF: If there exists a  $\phi \in (0, 1)$  such that  $\phi^*(X_1^H) < \phi \leq \phi^*(X_1^L)$  for  $F_0^u = \hat{F}_0^u$  (where  $\hat{F}_0^u$  and  $\phi^*$  are as defined in Lemmas A.2 and A.14), then B issuing secured debt would trigger acceleration if  $Q = L$ , deterring B from doing so (Lemma A.12), but not if  $Q = H$ , allowing B to do so. The Date 1 outcomes are efficient and thus renegotiation-proof, and B can raise  $I_0$  at Date 0 via unsecured debt with face value  $\hat{F}_0^u$ .

We therefore need only show that such a  $\phi$  exists. Then Lemma A.14 implies that  $\phi^*(X_1^H) < \phi^*(X_1^L)$ , we need only show that  $\phi^*(X_1^H) < 1$ . This follows from the fact that  $\hat{F}_0^u > X_0 + X_1^H - I_1/p$  by the hypothesis that  $X_1^L > X_1^H$ , Assumption 3, and the fact that  $\hat{F}_0^u \geq I_0/p$  (from the definition of  $\hat{F}_0^u$  in Lemma A.2).

Thus,  $\phi^*(X_1^H) = \frac{X_0 + X_1^H - I_1/p^2}{X_0 + X_1^H - I_1/p} < 1$ .  $\square$

LEMMA A.18: If  $X_1^L < X_1^H$  and  $p(X_0 + X_1^H) < \bar{F}_0^u + I_1$ , an efficient equilibrium does not exist.

PROOF: If  $p(X_0 + X_1^H) < \bar{F}_0^u + I_1$ , efficiency requires  $\phi \geq \phi^*(X_1^H)$  at  $F_1^s = I_1/p$  (Lemma A.14), and so  $\phi > \phi^*(X_1^L)$  as  $X_1^H > X_1^L$ . Hence, the covenant is irrelevant and efficiency cannot obtain (Assumption 3).

Now say that  $p(X_0 + X_1^H) \geq \bar{F}_0^u + I_1$ , which implies  $F_0^u < \bar{F}_0^u$ . Consider  $Q = H$ . In this case, B borrows with face value  $F_1^s \geq I_1$ . Acceleration is renegotiated

away and unsecured creditors get  $F_0^u$ . Now consider  $Q = L$ . If B has no incentive to invest or if investment would trigger acceleration, B will not invest (Lemma A.12) and unsecured investors get payoff  $p \min\{X_0, F_0^u\}$ . If B has an incentive to invest and if investing would not trigger acceleration, investment will be renegotiated away. Since B has full bargaining power, creditors' payoff equals what they would get under investment, that is,  $p \min\{X_0 + X_1^L - I_1/p, F_0^u\}$ . By condition (2), this is less than  $p \min\{X_0, F_0^u\}$ . Hence, unsecured creditors' expected payoff is at most  $qF_0^u + (1 - q)p \min\{X_0, F_0^u\} - I_0$ , which is negative since  $F_0^u < \bar{F}_0^u$ . That contradicts the premise that B funds Project 0 at Date 0.  $\square$

### Proof of Proposition 5

The result is a corollary of Propositions 1, 2, and 4 and their proofs.

## Appendix B: Robustness

Here we consider several generalizations of our model. We make two assumptions to simplify the analysis: (i) There is no renegotiation (other than waiving covenants) and (ii) covenant-protected debt is not paid in full following a secured debt issuance (so the issuance is in fact dilutive).

### A. Bankruptcy Due to Acceleration

So far we have assumed that a covenant-protected creditor can choose the fraction of debt it accelerates. Here we explore the case in which a creditor must accelerate either all of its debt or none of it. We show that our results do not change.

We must check that there is a level of covenant-protected debt  $F_0^c$  such that acceleration triggers bankruptcy when  $Q = H$  (so creditors do not accelerate) but not when  $Q = L$  (so they do), that is,

$$p(X_0 + X_1^H) - F_1^s < F_0^c \quad (\text{B1})$$

and

$$p(X_0 + X_1^L) - F_1^s \geq F_0^c. \quad (\text{B2})$$

These inequalities can hold whenever  $X_1^L \geq X_1^H$ , affirming the conclusion of Proposition 4.<sup>32</sup>

<sup>32</sup> For completeness, we should also check that the covenant-protected creditor does indeed choose not to accelerate if it triggers bankruptcy, in which case all unsecured debt is accelerated and therefore is *pari passu* (Section I.B). That condition is

$$\phi(p(X_0 + X_1^Q) - F_1^s) \leq \phi p(X_0 + X_1^Q - F_1^s). \quad (\text{B3})$$



### B. Imperfectly Correlated Cash Flows

So far we have assumed that projects are perfectly correlated. Here we show that this is not essential for our results (albeit under some stricter parameter restrictions).

Say that projects may be only imperfectly correlated—each succeeds with probability  $p$ , but the probability  $p'$  that both succeed can be less than  $p$ . In general, this complicates the analysis because there are four possible outcomes instead of two. To simplify, assume that the cost of Project 1 is sufficiently large that B can repay the debt used to finance it only if both projects succeed, so  $F_1^s \in [\max\{X_0, X_1^Q\}, X_0 + X_1^Q]$ . We also assume that if B uses secured debt at Date 0, its face value  $F_0^s$  is in this range.

Under these assumptions, the first-best is implementable via a mix of secured and unsecured debt at Date 0 whenever the face value of secured debt  $F_0^s$  is such that B can finance Project 1 if and only if it has positive value,

$$p'(X_0 + X_1^Q - F_0^s) \geq I_1, \quad (\text{B4})$$

or at Date 1 if  $Q = H$  but not if  $Q = L$ . This is the same condition as in the baseline model except  $p$  is replaced by  $p'$  (see equation (8)).

The first-best is implementable via a mix of covenant-protected and unprotected debt whenever there is a fraction  $\phi$  of covenant-protected debt such that the acceleration threat is credible following investment if and only if the investment has negative value, or

$$p(X_0 + X_1^Q) - F_1^s \geq \phi p'(X_0 + X_1^Q - F_1^s) \quad (\text{B5})$$

if  $Q = L$  but not if  $Q = H$ . This coincides with the condition (described in Section II.C) except  $p$  is replaced by  $p'$  in the event of continuation (of acceleration/liquidation).

Overall, the analysis here suggests that relaxing our assumption that projects are perfectly correlated does not change our qualitative results. It further suggests, however, that it could generate additional testable predictions. In the case analyzed here, it follows from  $p' > p$  that the fraction of secured debt needed to implement the first-best is higher than in the baseline and the fraction of covenant-protected debt is lower. (But we think a fuller analysis, without such stark simplifying assumptions, would be desirable before any actual empirical tests.)

### C. Asset Sale Discounts

So far we have assumed that selling assets destroys private benefits but not cash flows. We now consider an additional cost, for example, because it entails early termination or is organized hastily.

Say that the sale's proceeds are  $\lambda p(X_0 + X_1^Q)$  with  $\lambda < 1$ . The analysis of the debt structure without covenants is unchanged, as it does not involve asset

sales. That of the debt structure with covenants does change, however, because covenant-protected creditors now find it optimal to accelerate if

$$p\lambda(X_0 + X_1^Q) - F_1^s \geq \phi p(X_0 + X_1^Q - F_1^s). \quad (\text{B6})$$

This the same condition as in the baseline (Section II.C), but with the LHS multiplied by  $\lambda$ , reflecting the fact that the asset sale discount makes acceleration more costly.

Thus, including the discount does not change our qualitative results, but suggests an additional testable prediction: The higher the discount  $(1 - \lambda)$ , the lower  $\phi$  should be to induce creditors to accelerate optimally.

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