THE PARADOX OF PLEDGEABILITY*

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Abstract

We develop a model in which collateral serves to protect creditors from the claims of other creditors. We find that, paradoxically, borrowers rely most on collateral when pledgeability is high, because this is when it is easy to take on new debt, which dilutes existing creditors. Creditors thus require collateral for protection against dilution—there is a "collateral rat race." But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment—there is a "collateral overhang." Our results suggest that policies aimed at increasing the supply of collateral may backfire, triggering an inefficient collateral rat race. Likewise, upholding the absolute priority of secured debt can exacerbate the rat race.

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1 Introduction

Collateral matters.[1] In much of the finance literature, collateral matters because it mitigates enforcement frictions between borrowers and creditors, i.e. "collateral pledging makes up for a lack of pledgeable cash" (Tirole (2006), p. 169). But collateral also plays another role, emphasized in the law literature:[2] collateral matters because it mitigates enforcement frictions among creditors, i.e. "a secured transaction [is] the protection...against the claims of competing creditors" (Kronman and Jackson (1979), p. 1143). These two roles of collateral correspond to the two components of property rights which accrue to secured creditors upon default: the "right of access"—a creditor's right to seize collateral—and the "right of exclusion"—a creditor's right to stop other creditors from seizing collateral (e.g., Hart (1995), Segal and Whinston (2012)).

In this paper, we present a finance model based (solely) on the latter role. We find that borrowers rely on collateral when pledgeability is high, not low—collateral does not make up for a lack of pledgeable cash. The reason is that high pledgeability makes it easy to take on new debt, which dilutes existing creditors. This leads existing creditors to require collateral for protection against possible dilution by collateralized debt—there is a collateral rat race. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment—there is a collateral overhang. Further, greater availability of collateral can have adverse effects—it can trigger an inefficient collateral rat race. Hence, policies aiming to increase the supply of collateral, such as expanding the set of assets that can be used as collateral, may backfire. Likewise, upholding the absolute priority of secured debt could facilitate dilution rather than protect against it, triggering the rat race.

Model preview. A borrower, B, has two riskless projects, Project 0 and Project 1, to finance sequentially. B finances Project 0 by borrowing from one creditor, C_0. After Project 0 is underway, B can finance Project 1 by borrowing from another creditor, C_1. Project 0's NPV is positive, but Project 1's NPV, which is revealed after Project 0 is underway, may be positive or negative. Thus, it is efficient for B to undertake Project 0 and to undertake Project 1 only in the event that its NPV is positive.

B's borrowing capacity is constrained by two frictions. First, *pledgeability* is limited. Specifically, the total repayment from B to his creditors is limited to a fixed fraction θ of his projects' final value, representing, e.g., the liquidation value of the assets employed in the

[1]See, e.g., Aretz, Campello, and Marchica (2017), Benmelech and Bergman (2009, 2011), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014) for empirical evidence on the importance of collateral for borrowing.

[2]See, e.g., Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984, 1994, 1997).

projects. Second, contracts are *non-exclusive* in that when B takes on debt to C_0, he cannot commit not to take on new debt to C_1.[3]

In line with the law literature, we assume that collateral mitigates the non-exclusivity friction (but, for now, that it does not affect pledgeability). By borrowing secured, B "ring fences" his project(s) as collateral, protecting it from the claims of other creditors. Thus, if B borrows via secured (i.e. collateralized) debt, the secured creditor has an exclusive claim over the project(s) securing the debt. Indeed, legally, "[t]he absolute priority rule describes the basic order of payment in bankruptcy. Secured [i.e. collateralized] creditors get paid first, unsecured creditors get paid next" (Lubben (2016), p. 581).

Last, we assume that collateral cannot be state-contingent: a secured creditor always has an exclusive claim—it cannot be diluted in any state—whereas an unsecured creditor always has a non-exclusive claim—it can be diluted in any state. In that respect, collateral is only a coarse solution to the non-exclusivity problem.

Results preview. Our first main result is that, paradoxically, if pledgeability θ is sufficiently high, B may not be able to borrow from C_0 unsecured. To see why, suppose B finances Project 0 by borrowing from C_0 via *unsecured* debt. Because unsecured contracts are non-exclusive, B can approach another creditor, C_1, to finance Project 1. If B collateralizes both his projects to borrow from C_1, then C_1 is prioritized over C_0—the new secured debt dilutes the existing unsecured debt. As a result, C_0 may not lend unsecured in the first place. However, this dilution occurs only if C_1 is willing to finance Project 1, i.e. if the value of B's pledgeable payoff exceeds his funding needs. In summary, high pledgeability relaxes B's borrowing constraint with C_1, but it tightens his borrowing constraint with C_0. Hence, contrary to common intuition, high pledgeability undermines unsecured credit.

Our second main result is that, in anticipation of being diluted with new secured debt to C_1, the initial creditor C_0 will not lend unsecured, but only with collateral. There is a *collateral rat race*, by which collateralization is required as protection against future collateralization. In some circumstances, a mix of collateralized and uncollateralized debt gives rise to the first-best outcome, in which B can fund all (and only) positive NPV projects. Hence, our model also casts light on the coexistence of these two types of debt in borrowers' capital structure.

Our third main result is that if B borrows from C_0 via secured debt, B may be unable to fund Project 1, even when it has positive NPV. This is because collateralizing Project 0 "uses up" pledgeable collateral, making it difficult for B to borrow to finance Project 1. To curb

[3]Note that this assumption rules out covenants by which a borrower commits to one creditor not to borrow from new creditors in the future. As we discuss in Section 4, in reality, such covenants are rarely effective at preventing a borrower from borrowing secured from new creditors.

over-investment (by preventing inefficient dilution in some states), collateralization can thus also cause under-investment (by preventing efficient dilution in other states). In other words, collateralization effectively encumbers B's assets, i.e. it limits B's ability to use them to obtain funding to invest. This *collateral overhang* problem resonates with practitioners' intuition that "encumbered assets are generally not available to obtain...liquidity" (Deloitte Blogs (2014)).

Next, we enrich our model by supposing that only a fraction of a project can be collateralized. Some assets used for a project can be pledgeable—they can be seized in the future—but not collateralizable—they are hard to assign property rights to, e.g., they may not even exist at inception, but rather be built/acquired in the course of the project. Our fourth main result is that, although higher collateralizability can loosen borrowing constraints, it can also tighten them. This is because increasing collateralizability does *collateral damage:* it makes it easier to take on new debt from C_1, diluting C_0; this can trigger the collateral rat race. The more collateralizable Project 1 is, the more collateralizable Project 0 must be to provide protection against dilution. Hence, the more assets that can be used as collateral at Date 1, the more assets are required as collateral at Date 0—increasing the supply of collateral increases the demand for collateral. And, thus, higher collateralizability can exacerbate the collateral overhang.

We also extend the model so that collateral plays two roles. It mitigates enforcement problems among creditors—establishing exclusivity as in our baseline model—as well as between borrowers and creditors—increasing pledgeability as in most of the finance literature. Our fifth main result is that this classical role dominates for low pledgeability, when borrowers need collateral to get projects off the ground. But the new role we focus on dominates for high pledgeability, when creditors need collateral for protection against dilution.

Policy. Our analysis has implications for some public policies. First, some policies have aimed to increase the supply of collateral, deemed insufficient. For example, a number of countries expanded the set of movable/floating assets that can be used as collateral. To the same end, some central banks committed to lend against illiquid financial securities at a pre-specified rate and haircut.[4] Our results suggest that such policies may backfire, since increasing the supply of collateral may increase the demand for collateral, by triggering an inefficient rat race. Second, "[c]urrent law forces onto borrowers the power to defeat unsecured lenders by issuing secured debt" (Bjerre (1999), p. 308). Our analysis suggests that upholding the absolute priority of secured debt as such can lead to inefficient investment. This gives support to arguments advanced in the law literature against this absolute priority

[4]To increase the supply of financial collateral, The European Central Bank enacted its Long-term Refinancing Operation and the Reserve Bank of Australia its Committed Liquidity Facility.

rule (see Bjerre (1999) and Lubben (2016)).[5]

Applications. We model a new role of collateral: to establish exclusivity. This is likely its main role when dilution/non-exclusivity is the first-order friction. For example, non-exclusivity is especially relevant for financial firms that can have thousands of creditors. So is collateral. In fact, upwards of five trillion dollars of securities are pledged as collateral in interbank markets. Pledgeability is ostensibly high in these markets, due to strong creditor rights, effective legal enforcement, intense regulatory supervision, and developed record-keeping technologies. Hence, the reliance on collateral is not easily explained by the classical theory—i.e. collateral matters even when it is not necessary "to make up for a lack of pledgeable cash." Indeed, in the securities lending market, cash itself *is* the collateral—borrowers pledge cash to borrow securities—and, even in the repo market, the securities used as collateral are typically so liquid that they are referred to as "cash equivalents." Our model suggests these borrowers rely on collateral because they need to establish priority.

Generally, our model applies to situations in which borrowers can use secured debt to dilute existing debt, a ubiquitous problem of unsecured debt. For example, firms in distress can collateralize assets to meet their operating costs, thereby using dilution to gamble for resurrection. Likewise, households collateralize assets to get emergency liquidity, e.g., by refinancing their mortgages. Borrowers can also collateralize assets implicitly via trade credit or leasing (borrowing secured to finance an asset purchase is almost equivalent to leasing the asset). In our model, this can trigger the collateral rat race. Hence, the ability to collateralize assets does not always benefit borrowers. Again, this contrasts with received theory, but resonates well with practice. Indeed, lawyers observe that "borrowers would prefer to give up that power [to use collateral] in order to protect their unsecured lenders from the corresponding threat" (Bjerre (1999), p. 308). I.e., sometimes borrowers are better off without collateral.

Evidence. Degryse, Ioannidou, and von Schedvin (2016) focus explicitly on non-exclusivity in credit. They analyze the effect of a borrower breaking up an exclusive relationship with its existing creditor by borrowing from a new creditor. They find the existing creditor becomes less willing to lend unsecured, but not less willing to lend secured. Our paradox of pledgeability provides an explanation for this result: a borrower's ability to borrow from a new creditor undermines his ability to borrow unsecured from existing creditors.

Many empirical papers document that increasing creditor/collateral rights can increase lending and investment. In our model, higher pledgeability/collateralizability can have this

[5]Bolton and Oehmke (2015) make a related argument for derivatives. They point out that the priority for derivatives, such as safe harbors for derivatives collateral, can increase a borrower's total cost of funding and decrease overall efficiency.

effect. It can be the beneficial result of loosening financial constraints, as in the received theory. However, it can also be the detrimental result of diluting existing debt. Hence, one should be careful about drawing welfare conclusions from these findings.

In contrast to these findings, Vig (2013) analyzes a reform that made it easier for secured creditors to seize assets from defaulting borrowers, which, in our model, corresponds to an increase in collateralizability. He finds that lending went down, in contrast to standard theories, but in line with our collateral damage result: increasing collateralizability can make dilution easier and undermine unsecured credit.

Like us, Haselmann, Pistor, and Vig (2010) stress the distinction between collateralizability (that we denote μ) and pledgeability (that we denote θ). They find that μ, captured by collateral law, matters more for credit supply than θ, captured by bankruptcy law. This suggests that the non-exclusivity frictions, mitigated by collateral, may be as important as ex post limits to enforcement (pledgeability frictions), mitigated by bankruptcy.

Proxies. In line with the legal proxies in Haselmann, Pistor, and Vig (2010), collateralizability μ corresponds to property/collateral rights and pledgeability θ corresponds to creditor rights. Asset turnover may be another (inverse) proxy for μ, as it is hard for a borrower to use assets as collateral if they are not yet in his possession or they will not stay in his possession for very long.[6] And another way to capture θ is with asset tangibility, since more tangible assets are easier to repossess (see, e.g., Rampini and Viswanathan (2013)).

To test our model, it is also useful to proxy for non-exclusivity, i.e. for a borrower's access to multiple creditors. One simple proxy is having multiple creditors already. But not all single-creditor borrowers have the same access to multiple creditors. A more robust proxy could exploit the liberalization in bank branching regulation, which gives borrowers access to more potential creditors. Size and transparency could also proxy for exclusivity, since small and opaque borrowers are likely to be captive to their incumbent creditors.

New predictions. Our analysis suggests six, as yet untested, predictions specific to collateral establishing exclusivity.

1. Increasing pledgeability θ leads borrowers to increase the proportion of secured debt. The reason is that increasing θ can trigger the collateral rat race, making borrowers more reliant on secured debt (cf. Proposition 2). Hence borrowers increase their secured debt more than their unsecured debt.

2. Increasing collateralizability μ leads borrowers to increase the proportion of secured debt. The reason is that, likewise, increasing μ can trigger the collateral rat race (cf.

[6]Sometimes a borrower can still collateralize short-term inventories via a floating charge, by which he collateralizes a fund of changing assets.

Proposition 4).

3. Increasing θ leads borrowers first to increase and then decrease their leverage. The reason is that for low θ, increasing θ loosens financial constraints (the standard effect of pledgeability), but for high θ, it tightens them, by making dilution easier (cf. Proposition 5).

4. Leverage and investment's sensitivity to μ increases with θ. The reason is that for low θ, borrowers are always constrained, whereas for high θ, they are constrained for low μ but not for high μ (cf. Proposition 4).

5. The more severe is the non-exclusivity friction, the more sensitive leverage is to θ. Indeed, borrowers with non-exclusive relationships could be rationed due to the collateral overhang, and hence increase their leverage less than those in exclusive relationships.

6. The more severe is the non-exclusivity friction, the more sensitive leverage is to μ. The reason is that μ affects only non-exclusivity and thus matters only to borrowers in non-exclusive relationships.

Contribution to the literature. To our knowledge, our model is the first to focus on the role that collateral can play in mitigating non-exclusivity, arguably its role legally. That said, Ayotte and Bolton (2011) also adopt a legal definition of a secured creditor's property rights, but they focus on when these rights should be enforced given that other creditors may be unaware of them. Bolton and Oehmke (2015) also analyze the priority of some claims over others, but they focus on when derivatives should be privileged over other liabilities.[7]

Some of the mechanisms behind our results have parallels in finance papers that do not study collateral. Our "paradox of pledgeability"—higher pledgeability undermines a borrower's ability to commit to future borrowing decisions—is a liabilities-side analog of Myers and Rajan's (1998) asset-side "paradox of liquidity"—higher asset liquidity undermines a borrower's ability to commit to future investment decisions (since it makes liquidating assets for expropriation more attractive).[8] However, in our model, the borrower always wants to dilute, but cannot when pledgeability is low because creditors will not lend. In

[7]A number of finance theory papers study non-exclusive contracting but do not focus on collateral, e.g., Acharya and Bisin (2014), Admati, DeMarzo, Hellwig, and Pfleiderer (2013), Attar, Casamatta, Chassagnon, and Décamps (2015, 2017), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and DeMarzo (1992), DeMarzo and He (2016), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).

[8]Donaldson and Micheler (2016) argue that higher pledgeability can also paradoxically foster systemic risk, because it leads borrowers to favor non-resaleable over resaleable debt instruments (e.g., repos over bonds).

contrast, in Myers and Rajan (1998), the borrower does not always want to liquidate, and chooses not to when liquidity is low because it is not desirable.

In our "collateral rat race" result, collateral plays a similar role to short maturity in Brunnermeier and Oehmke's (2013) "maturity rat race." In our model, unlike in theirs, too much protection against dilution can be inefficient, because it can induce the "collateral overhang."

This collateral-overhang problem bears some similarity to Myers's (1977) "debt-overhang problem," since debt in place prevents a borrower from financing positive-NPV projects. However, in the debt-overhang problem, a borrower will not raise capital because this would subsidize existing debt, whereas in the collateral-overhang problem, he cannot raise capital because existing debt is collateralized to prevent this. Further, the collateral-overhang problem arises even when the debt-overhang problem does not, i.e. when existing debt is riskless or can be renegotiated.

In our analysis of "collateral damage," we distinguish between pledgeable assets—those which creditors can seize ex post—and collateralizable assets—those which creditors can obtain property rights to ex ante. To our knowledge, this distinction is new to the theory literature, and hence so are the results that it generates.

Layout. The paper proceeds as follows. Section 2 presents the model. Section 3 contains the main results. Section 4 discusses the contracting environment. Section 5 concludes. The Appendix contains all proofs.

2 Model

2.1 Players and Projects

There is one good called cash, which is the input of production, the output of production, and the consumption good. A borrower B lives for three dates, $t \in \{0, 1, 2\}$, and consumes at Date 2. B has no cash, but has access to two investment projects, Project 0 at Date 0 and Project 1 at Date 1. Both projects are riskless and pay off at Date 2, but the payoff of Project 1 is revealed only at Date 1. Specifically, Project 0 costs I_0 at Date 0 and pays off X_0 at Date 2. At Date 1 there are two states, $s \in \{L, H\}$, with $p := \mathbb{P}[s = H]$. In state s, Project 1 costs I_1^s at Date 1 and pays off X_1^s at Date 2. Everyone is risk neutral and there is no discounting or asymmetric information.

B can fund his projects by borrowing I_0 at Date 0 and I_1^s in state s at Date 1 from competitive credit markets: we assume that B makes a take-it-or-leave-it offer to borrow from creditor C_t at Date $t \in \{0, 1\}$.

2.2 Pledgeability and Collateralizability

B promises to repay his creditor(s) under two frictions.

First, pledgeability is limited in that B may divert a fraction $(1-\theta)$ of projects' payoffs, leaving only the pledgeable fraction θ for his creditors. θ is the portion of a project's final value that creditors can seize. For example, θX could represent the value of the assets used in a project and $(1-\theta)X$ could represent its terminal cash flow.[9]

Second, contracts are non-exclusive in that if B borrows from C_0 at Date 0, he cannot commit not to borrow from C_1 at Date 1, potentially diluting C_0's initial claim. (This rules out covenants that prevent future borrowing, as we discuss in detail below (see Section 4).)

The role of collateral in our model is to mitigate the second friction: if B chooses to collateralize (or "secure") a fraction σ of a project with payoff X, a creditor gets the exclusive right to that fraction of the project's pledgeable payoff, i.e. absolute priority over $\sigma\theta X$.[10,11] However, as we describe next, we assume that σ does not depend on the state. Hence, in that sense, collateral is only a "coarse" solution to the non-exclusivity problem, as we discuss in Section 4.

2.3 Borrowing Instruments

At Date t, B borrows the cost of Project t from C_t against the promise to repay the fixed amount F_t at Date 2. This promise can be secured, i.e. collateralized, or unsecured. If B chooses to collateralize a fraction σ_0 of Project 0 to C_0, then C_0 has priority over $\sigma_0\theta X_0$. This fraction of Project 0 cannot be collateralized again to C_1. However, anything that B has not collateralized to C_0 can be partially collateralized to C_1. Thus, B can choose to give C_1 a senior claim on (at most) the fraction $(1-\sigma_0)$ of Project 0 and all of Project 1.

If there are multiple unsecured creditors, we assume that they are on equal footing in the event of B's default at Date 2. This is consistent with their pari passu legal treatment.

[9] Beyond this interpretation, there is no formal difference between assets and cash flows. We choose a single-good set-up to focus on how collateral is used to establish priority, rather than to increase pledgeability, as in much of the finance literature (see, however, Subsection 3.7 and Subsection 3.8). In that literature, there are typically two goods with different intrinsic properties that determine which one serves as collateral. For example, a pledgeable good (e.g., physical capital) could be used as collateral to borrow a "divertable" good (e.g., cash). In our single-good model, in contrast, whether a good serves as collateral depends only on the borrower's choice of debt instrument, not on the good itself.

Our perspective can cast light on situations in which whether a good is used as collateral seems not to depend on its intrinsic properties. For example, borrowers use securities as collateral to borrow cash in the repo market and use cash as collateral to borrow securities in the securities lending market. Borrowers are not using a "pledgeable" good as collateral to borrow a "divertable" good; rather, we suggest, they are using it as collateral to establish priority.

[10] For a similar concept of securing assets away from third parties see Kiyotaki and Moore (2000, 2001).

[11] In Subsection 3.7, we consider the case in which only part of a project is collateralizable, i.e. there is an upper bound on σ.

We capture this by having unsecured creditors fifty-fifty Nash bargain at Date 2 over the residual value after the secured debt is paid.[12]

Our results are not sensitive to the fine details of the contracting environment. None of them depends on whether Date-2 repayments can be state-contingent, and only the "collateral damage" results depend on the priority rule among unsecured creditors (Subsection 3.7).[13] Rather, our main results depend only on the assumptions that (i) B cannot commit not to collateralize in the future, (ii) secured debt is treated as senior, and (iii) B cannot make the fraction σ_0 of Project 0 he collateralizes depend on the Date-1 state. These assumptions reflect the real-world constraints that current law imposes on borrowers, as we explain in detail in Section 4.

2.4 Payoffs

We now give the players' terminal payoffs. First, define the indicator variable $\mathbb{1}_t$ as follows:

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken,} \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Thus, the total payoff from all projects undertaken is given by

$$W := \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1. \tag{2}$$

B's payoff is the sum of two terms: (i) the non-pledgeable part of the payoff from the project(s) and (ii) the residual of the pledgeable part of the payoff after making repayments, i.e. B's payoff is $(1-\theta)W + \max\{\theta W - F_0 - F_1, 0\}$. If B does not default—i.e. $F_0 + F_1 \leq \theta W$—then creditor C_t gets F_t. If B does default—i.e. $F_0 + F_1 > \theta W$—then C_0 and C_1 divide θW according to priority.

[12] As Bjerre (1999) puts it, "the 'pari passu principle,' provides that unsecured creditors rank equally with each other in right to payment, regardless of the temporal order in which they extend credit" (p. 309). In practice, unsecured creditors are prioritized roughly according to the order in which they alert the court of a borrower's default, i.e. the "first to file or perfect" is paid first (see, e.g., Picker (1999)). Our assumption of fifty-fifty bargaining is akin to assuming that creditors are equally likely to win this race to alert the court. Our results also hold for general bargaining power. (For an influence-cost-based model that endogenizes creditors' bargaining positions given default, see Welch (1997).)

[13] That said, this priority rule among unsecured creditors is realistic (cf. footnote 12). Moreover, contracting on seniority may be difficult even in theory. The reason is that even if unsecured creditors contract on seniority, they may enter into conflicting contracts—e.g., each creditor has a contract that says it is senior. Registries for secured debt exist to solve this problem. See Ayotte and Bolton (2011) for a model of priority based on the costs of checking for conflicting contracts.

2.5 Assumptions

We impose several restrictions on parameters.

Assumption 1. The pledgeable payoff of Project 0 in state L alone is worth more than I_0: $I_0 < (1-p)\theta X_0$.

This implies C_0 lends as long as it knows it will not be diluted in state L (even if it will be diluted in state H; see Lemma 2). This also implies that it is efficient to undertake Project 0, i.e. that $I_0 < X_0$.

Assumption 2. Project 1 has positive NPV in state H but negative NPV in state L: $I_1^H < X_1^H$ but $I_1^L > X_1^L$.

This implies it is efficient to undertake Project 1 in state H only.

Assumption 3. In both states, $s \in \{L, H\}$, the combined pledgeable cash flow from Project 0 and Project 1 is less than the investment cost: $\theta(X_0 + X_1^s) < I_0 + I_1^s$ for $s \in \{L, H\}$.

This implies that the limited pledgeability friction is severe enough that it may prevent B from investing even when it is efficient (i.e. in state H).

Assumption 4. The non-pledgeable payoff of Project 1 is not too small: $(1-\theta)X_1^L > \theta X_0 - I_0$.

This more technical condition ensures that the payoff of Project 1 is always large enough that B has the incentive to undertake it (Lemma 1).[14]

Assumption 5. The cost of Project 1 is not too large: $I_1^H < \theta(X_0 + X_1^H)$.

This technical condition ensures that the cost of Project 1 is not so large that B can not borrow from C_1 to finance it even if he collateralizes both Project 0 and Project 1 to C_0.

These assumptions serve to streamline the analysis by restricting attention to relevant cases. They ensure that non-exclusivity, the basic friction in our model, bites: B must dilute C_0 to invest efficiently in state H (Assumption 3), but also has incentive to dilute C_0 to invest inefficiently in state L (Assumption 4).

3 Results

In this section, we analyze the model's subgame perfect equilibrium outcomes.

[14] Alternatively, we could assume that B gets private benefits from empire building and, therefore, always has the incentive to undertake Project 1, regardless of its NPV. In that case this assumption is unnecessary.

3.1 Preliminaries

The following two lemmata are useful. They follow quickly from the assumptions.

LEMMA 1. *B always borrows if it is feasible.*

Notably, this result implies that as long as C_1 is willing to lend at Date 1, B borrows to invest in Project 1, even in state L, when it has negative NPV. The reason is that borrowing from C_1 may dilute C_0's debt, subsidizing B's investment. Assumption 4 implies this subsidy is large enough that B always wants to borrow.

LEMMA 2. *If B can commit not to borrow from C_1 in state L at Date 1, B can borrow from C_0 at Date 0.*

This result follows immediately from Assumption 1 that B's pledgeable payoff in state L alone is worth more than I_0.

3.2 First Best

In the first-best outcome, all positive NPV projects are undertaken.

LEMMA 3. *The first-best outcome is to undertake Project 0 at Date 0 and Project 1 at Date 1 if and only if $s = H$.*

3.3 Paradox of Pledgeability

We find that if θ is low, B borrows unsecured at Date 0 and the first best is attained. But, in contrast, if θ is high, B cannot borrow unsecured at Date 0—there is a paradox of pledgeability.

PROPOSITION 1. (PARADOX OF PLEDGEABILITY) *Define*

$$\theta^* := \frac{I_1^L}{X_0 + X_1^L}. \tag{3}$$

If $\theta < \theta^*$, C_0 lends unsecured and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not borrow in state L.

If $\theta \geq \theta^*$, C_0 does not lend unsecured.

Low pledgeability prevents B from borrowing from C_1 in state L. Thus, C_0 is not worried about being diluted in state L. Since C_0 is repaid in state L, it lends unsecured at Date 0 even though it will be diluted in state H (Lemma 2). This makes it easier for B to borrow from C_1 in state H and invest efficiently.[15]

Conversely, higher pledgeability allows B to pledge more to C_1, making C_1 more willing to lend.[16] Indeed, even though B will be unable to repay both creditors in full (Assumption 3), C_1 is still willing to lend via secured debt, since this new debt to C_1 is senior to B's existing debt to C_0. However, anticipating this dilution, C_0 refuses to lend in the first place—higher pledgeability makes it easier to borrow at Date 1 and, hence, paradoxically, makes it harder to borrow at Date 0.

Note that our result does *not* rely on the assumption that the repayment F_t is not contingent on the state s, since contingent contracts do not help B commit not to dilute C_0 in state L. We spell this out in the Appendix after the proof of the proposition.

Of course, in general, very low pledgeability would prevent borrowing at Date 0. This inefficient outcome is ruled out by Assumption 1 (see, however, Subsection 3.8). And, further, very high pledgeability can also restore efficiency. But this is ruled out by Assumption 3.

3.4 Collateral Rat Race

We now show that collateralization at Date 0 can protect from dilution at Date 1 (Proposition 1). I.e. collateralization today can preempt future collateralization. Indeed, an appropriate level of collateralization may yield the first-best outcome.

Proposition 2. (Rat race) *Define*

$$I_1^* := I_1^L + \theta\big(X_1^H - X_1^L\big). \tag{4}$$

If $I_1^H < I_1^*$, B borrows (partially) secured from C_0 and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not borrow in state L.

If funding needs are small enough in state H ($I_1^H < I_1^*$), the appropriate mix of secured and unsecured debt at Date 0 leads to efficient investment at Date 1, since B is too constrained to borrow from C_1 in state L but not in state H. In other words, if B takes on partially secured debt to C_0, he can partially dilute it. This partial dilution allows him to borrow enough to fund Project 1 in state H (efficiently), but not in state L (inefficiently).

[15] Optimal "dilutable debt" also appears in Diamond (1993), Donaldson and Piacentino (2017), Hart and Moore (1995), and Stulz and Johnson (1985).

[16] This intuition depends on the fixed-scale nature of B's projects. If B cannot raise enough capital to undertake a project, he does not dilute existing debt. However, the same intuition would result if B had a fixed cost of starting up a scaleable project or a fixed cost of raising capital. More importantly, our key insight is that B needs collateral even when pledgeability is high. This is the case even with perfectly scaleable projects with zero start-up cost. In that case, B just relies on collateral to protect against dilution for low pledgeability too. Thanks to Adriano Rampini for a discussion illustrating this point.

This result says that although for high pledgeability B cannot invest efficiently if he borrows unsecured at Date 0 (Proposition 1), he can if he uses the appropriate amount of collateral. The reason B needs to use collateral at Date 0 is that he cannot commit not to use it at Date 1—there is a collateral rat race in which creditors require collateral today to protect against dilution with collateral in the future.

This resonates with the work of legal scholars who have also observed that collateral is necessary to "protect lenders against dilution [with] secured debt" (Schwartz (1997), p. 1397) given that "[l]ate-arriving secured creditors can leapfrog earlier unsecured creditors, redistributing value to the benefit of the issuer and the secured creditor but to the detriment of unsecured creditors" (Listokin (2008), p. 1039). Although this use of collateral can help to restore efficiency by protecting creditors against inefficient dilution, as above, it can also create inefficiencies, by preventing efficient borrowing, as we show next.

3.5 Collateral Overhang

While collateralization prevents B from borrowing when he should not, it can also prevent B from borrowing when he should. In other words, collateralization can encumber assets, leading to a collateral-overhang problem.

PROPOSITION 3. (COLLATERAL OVERHANG) *If $I_1^H \geq I_1^*$, B borrows from C_0 at Date 0 secured by a fraction of Project 0 and B cannot borrow from C_1 at Date 1 in either state. Hence, there is inefficient underinvestment in state H.*

If funding needs are large enough in state H ($I_1^H \geq I_1^*$), by committing not to borrow in state L, B commits not to borrow in state H. This points to a downside of collateralization: whereas it prevents B from diluting C_0 to fund an inefficient investment in state L, it also prevents him from diluting C_0 in state H to fund an efficient investment—collateralization encumbers B's assets. Thus, the risk of future collateralization may lead to inefficient preemptive collateralization. Further, ex interim renegotiation cannot resolve this inefficiency.

COROLLARY 1. *The equilibrium debt contract is renegotiation proof. I.e., B, C_0, and C_1 cannot renegotiate to undertake Project 1 in state H and thereby avoid the collateral overhang.*

Collateralization leads to inefficient asset encumbrance that cannot be renegotiated away. This is because limited pledgeability implies that the pledgeable payoff is insufficient to compensate C_0.[17]

[17] This finding that the "collateral overhang" of secured credit cannot be resolved by renegotiation complements Bhattacharya and Faure-Grimaud's (2001) finding that when a firm's investments are non-contractible, renegotiation between borrowers and creditors may not resolve the debt-overhang problem.

This result underscores that B must dilute C_0's debt in state H to borrow and invest efficiently in Project 1. It follows that the collateral overhang is not reliant on our assumption that debt matures at Date 2, i.e. short-term debt does not help. The reason is that if C_0 lends to B via short-term debt, B has no debt in place at Date 1. And, without diluting debt in place, B cannot borrow enough to invest in Project 1 in state H. We show this formally in the Appendix after the proof of the corollary.

Our results so far are closely in line with practitioners' intuition that "asset encumbrance not only poses risks to unsecured creditors"—collateralization dilutes unsecured creditors—"but also has wider...implications since encumbered assets are generally not available to obtain...liquidity"—collateralization leads to the collateral overhang (Deloitte Blogs (2014)). In other words, secured debt in place prevents a borrower from financing positive-NPV projects.

This is also the case in Myers's (1977) "debt-overhang problem," but for a different reason. In the debt overhang problem, a borrower prefers not to raise capital because the benefits of a new investment go to his existing creditor. In the collateral-overhang problem, the incentives of the borrower and the creditor are reversed: the borrower would prefer to raise capital because the benefits of new investment go to him at the expense of the creditor, but debt is collateralized precisely to stop him from doing so. Further, the debt-overhang problem does not arise if either debt in place can be renegotiated (allowing existing creditors to share the benefits of new investments) or if it is riskless (implying existing creditors get none of the benefits of new investments, since they are already repaid in full). In contrast, the collateral-overhang problem arises even though debt in place both can be renegotiated and is riskless.

The collateral overhang arises only for sufficiently high pledgeability, when the first best cannot be attained with unsecured debt (Proposition 1). However, increasing pledgeability even further can mitigate the collateral overhang. As long as increasing θ loosens B's borrowing constraint in state H more than his borrowing constraint in state L, it can facilitate efficient investment, solving the collateral-overhang problem.

COROLLARY 2. *Suppose $X_1^H > X_1^L$ and $I_1^H > I_1^L$ and define*

$$\theta^{**} := \frac{I_1^H - I_1^L}{X_1^H - X_1^L}. \tag{5}$$

If $\theta \geq \theta^{**}$, the first best is attained as in Proposition 2. There is no collateral overhang.

3.6 Equilibrium Characterization

The results so far imply the following equilibrium characterization.

COROLLARY 3. *The equilibrium outcome is as follows.*

- $\theta < \theta^*$: *at Date 0, B borrows unsecured with face value*

$$F_0 := \frac{I_0 - p\big(\theta(X_0 + X_1^H) - I_1^H\big)}{1-p}; \tag{6}$$

  at Date 1 in state H, B borrows secured with face value I_1^H*; and at Date 1 in state L, B does not borrow. The first best is attained.*

- $\theta \geq \theta^*$ *and* $I_1^H < I_1^*$: *B borrows partially secured with face value* F_0 *(equation (6)); at Date 1 in state H, B borrows secured with face value* I_1^H*; and at Date 1 in state L, B does not borrow. The first best is attained.*

- $\theta \geq \theta^*$ *and* $I_1^H \geq I_1^*$: *B borrows secured with face value* I_0*; at Date 1, B does not borrow in state H or state L. The first best is not attained due to the collateral overhang.*

This result suggests that borrowers may need collateral even when pledgeability is high. As discussed in the Introduction, this contrasts with received theories, but still resonates with practice. Indeed, some of the worlds' most developed debt markets rely heavily on collateral.[18]

3.7 Collateralizability and Collateral Damage

So far, we have assumed that all pledgeable assets can serve as collateral.

In reality, however, some assets may be pledgeable—they can be seized in the future—but not collateralizable—they are hard to assign property rights to. For instance, they may not even exist at inception, but rather be built or acquired in the course of the project. Also, property rights on some existing assets, such as intellectual property, may be difficult to define legally. How do pledgeability and collateralizability interact?

To address this question, we extend the model by assuming that B can collateralize at most a fraction μ_t of Project t at Date t, so B can divert $(1-\theta)X_t$ and collateralize $\theta\mu_t X_t$,

[18] The assumptions in Subsection 2.5 do impose one limit on interpretation here. Given Assumption 5 and the condition of Proposition 3, there are no fixed parameters p, I_0, X_0, I_1^H, X_1^H, I_1^L, and X_1^L such that there is efficient investment with unsecured debt for low θ and inefficient underinvestment with secured debt (collateral overhang) for high θ.

but $\theta(1-\mu_t)X_t$ is neither divertable nor collateralizable. Since different projects may employ different types of assets, μ_t depends on the project.

We find that higher collateralizability can loosen borrowing constraints to help finance efficient investments. However, it can also increase the risk of dilution, and hence tighten borrowing constraints.

LEMMA 4. *Suppose B collateralizes a fraction σ_0 of Project 0 at Date 0. If*

$$\mu_1 \geq \frac{2I_1^s - \theta(1-\sigma_0)X_0}{\theta X_1^s} - 1, \tag{7}$$

B invests in Project 1 in state s (and C_0 is diluted).

This result implies that high collateralizability μ_1 can do damage: because it makes it easier for B to take on new debt to C_1, diluting C_0, it can lead C_0 to require collateral as protection. In other words, it can trigger a collateral rat race. This can lead to a collateral overhang.

PROPOSITION 4. (COLLATERAL DAMAGE) *Define*

$$\mu_1^* := \frac{2I_1^L - (1-\mu_0)\theta X_0}{\theta X_1^L} - 1 \tag{8}$$

and suppose p is not too large. If $\mu_1 \geq \mu_1^*$, B does not invest at Date 0 or Date 1.

This lower bound μ_1^* on μ_1 implies that higher collateralizability can have adverse effects. The possibility of dilution at Date 1 can make it impossible for B to borrow at Date 0, since Project 0 may not be sufficiently collateralizable to protect C_0 against dilution. Further, observe that μ_1^* is increasing in μ_0. In other words, the more collateralizable Project 1 is, the more collateralizable Project 0 needs to be to offer C_0 the necessary protection.

This leads to the next corollary, which says that the demand for collateral at Date 0 may be increasing in the supply of collateral at Date 1.

COROLLARY 4. *Let σ_0^* denote C_0's demand for collateral, i.e. the smallest amount of collateral B can secure to C_0 so that C_1 prefers not to lend to B in state L:*

$$\sigma_0^* = \inf\left\{\sigma_0 \;\middle|\; \mu_1\theta X_1^L + \frac{1}{2}\Big((1-\sigma_0)\theta X_0 + (1-\mu_1)\theta X_1^L\Big) < I_1^L\right\}. \tag{9}$$

C_0's demand for collateral σ_0^* is increasing in Project 1's collateralizability μ_1.

This result has policy implications. Indeed, recently, governments have been "manufacturing quality collateral," because "there's still not enough of the quality stuff to go around...as quality collateral becomes impossible to find...[t]he crunch has further been heightened by the general trend towards collateralised lending and funding" (Kaminska (2011)). For example, several countries recently expanded the set of movable assets that can be used as collateral.[19] Moreover, in 2005, repo transactions backed by some assets became super senior in bankruptcy. In the context of our model, this corresponds to an increase in collateralizability. Note that it did not necessarily affect pledgeability: repo borrowers found it easier to assign assets as collateral to specific repo creditors, but did not find these assets any harder to divert.

Despite this effective increase in the supply of collateral, markets perceived a shortage of collateral. This is consistent with our idea that collateral supply creates collateral demand. As Caballero (2006) puts it, "The world has a shortage of financial assets. Asset supply is having a hard time keeping up with the global demand for...collateral" (p. 272); see also Di Maggio and Tahbaz-Salehi (2015).

3.8 The Two Roles of Collateral

So far, we have abstracted from how collateral mitigates enforcement problems between borrowers and creditors to focus on how it mitigates them among creditors. In reality, secured creditors have both (i) the "right to use" collateral—i.e. to seize the assets used as collateral—and (ii) the "right to exclude" others from collateral—i.e. to stop others from seizing the assets used as collateral (see, e.g., Hart (1995), Segal and Whinston (2012)). We now discuss an extension in which collateral plays both roles. We show that the "right to use" dominates for low pledgeability, whereas the "right to exclude" dominates for high pledgeability.

Here, we assume that the fraction of a project that is pledgeable depends on whether debt is secured or not: it is $\theta_c := c\theta$ if B borrows collateralized and $\theta_u := u\theta$ if B borrows unsecured, as in the corporate finance literature. We assume not only that collateralization establishes exclusivity, as in the baseline model, but also that collateralization increases pledgeability, i.e. $c > u$. We focus on the case in which B always has sufficient pledgeable cash flow to fund Project 0 via collateralized debt, i.e. $\theta_c X_0 > I_0$ and Assumption 5 holds with $\theta = \theta_c$, i.e. $I_1^H < \theta_c(X_0 + X_1^H)$.

Proposition 5. *The ability to borrow unsecured is hump-shaped in pledgeability, so increas-*

[19] A number of European countries recently allowed movable assets to be used as collateral for the first time (see Calomiris, Larrain, Liberti, and Sturgess (2017), Campello and Larrain (2016), Cerqueiro, Ongena, and Roszbach (2016), and Thell (2017)) as did Zimbabwe, where cows, sheep, and goats used as collateral are now recorded in a registrar at the central bank (Hawkins and Cotterill (2017)).

ing θ helps for small θ but hurts for high θ. More precisely, define

$$\theta_u^* := \frac{I_0}{uX_0} \quad \text{and} \quad \theta_c^* := \frac{I_1^L}{c(X_0 + X_1^L)}. \tag{10}$$

If $\theta \in \left[\theta_u^*, \theta_c^*\right)$, C_0 lends unsecured and the first best is attained, i.e. B borrows (secured) from C_1 in state H and does not in state L.

If $\theta \notin \left[\theta_u^*, \theta_c^*\right)$, C_0 does not lend unsecured.

For low θ, B cannot borrow unsecured from C_0, but must use collateral to increase his pledgeable payoff. For high θ, B also cannot borrow unsecured from C_0, but must use collateral to protect C_0 from dilution.

As in the baseline model, borrowing secured at Date 0 can lead to inefficient underinvestment at Date 1 due to the collateral overhang.

Corollary 5. *Define*

$$I_c^* := I_1^L + \theta_c\big(X_1^H - X_1^L\big). \tag{11}$$

If $I_1^H \geq I_c^*$, no equilibrium exists in which the first best is attained.

4 Discussion of Contracting Environment and Covenants

As we touched on in Subsection 2.3, the critical contracting assumptions are that (i) borrowers cannot commit not to collateralize in the future, (ii) secured debt is treated as senior, and (iii) borrowers cannot make collateralization contingent on future events. Here we argue that these assumptions reflect reality.

Commitment not to collateralize/Negative pledge covenants. We have assumed that contracts do not include covenants restricting future secured borrowing. If they were perfectly enforced, such covenants could protect creditors against dilution, hence limiting the need for collateral to establish exclusivity/priority. Although such covenants exist, they are relatively ineffective in practice. This is because, whereas a secured creditor holds a claim against other creditors (via a property right), an unsecured creditor holds a claim against only the borrower (via a promissory right; cf. Ayotte and Bolton (2011)). Thus, an unsecured creditor cannot recover collateral that has been seized by a secured creditor, even if the secured creditor violated a covenant. Bjerre (1999) describes these legal restrictions as follows:

> the negative pledge covenant [is a covenant] by which a borrower promises its lender that it will not grant security interests to other lenders. These covenants

> are common in unsecured loan agreements because they address one of the most fundamental concerns of the unsecured lender: that the borrower's assets will become unavailable to repay the loan, because the borrower will have both granted a security interest in those assets to a second lender and dissipated the proceeds of the second loan. Unfortunately, negative pledge covenants' prohibition of such conduct may be of little practical comfort, because as a general matter they are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered (pp. 306–307).

The effectiveness of negative pledge covenants in bankruptcy is especially limited for repo and derivatives liabilities, since these contracts are exempt from bankruptcy stays—i.e. creditors can liquidate collateral without the approval of the bankruptcy court, making it difficult or impossible for any third party to enforce a claim to the collateral.

Negative pledge covenants may still be useful outside bankruptcy. Since their violation constitutes a default, borrowers may adhere to the terms of covenants to avoid a default.[20] However, this may be insufficient to prevent a borrower from taking on additional debt in general. For example, a borrower in financial distress is likely to default anyway and may therefore be willing to violate such covenants to gamble for resurrection by taking on new debt. More generally, it can be difficult to verify that a solvent firm has violated a covenant.

Covenants are especially difficult to monitor/enforce for complex firms, notably banks, that may have thousands of counterparties. Indeed, banks effectively do not have to disclose their short-term borrowing:

> There are no specific MD&A requirements to disclose intra-period short-term borrowing amounts, except for [some] bank holding companies [that must] disclose on an annual basis the average, maximum month-end and period-end amounts of short-term borrowings (Ernst & Young (2010)).

There is a another reason that banks in particular may not be able to commit not to dilute existing debt with new debt: the very business of banking constitutes maturity and size transformation, which requires frequent short-term borrowing from many creditors. If a bank agrees to covenants that restrict its ability to borrow in the future, it could undermine its ability to engage in these banking activities (Bolton and Oehmke (2015)). This emphasizes that non-exclusive contracting is an especially important friction for banks and, therefore, it

[20] Other theory papers have shown how such covenants can mitigate incentive problems in some contexts. E.g., Rajan and Winton (1995) show that they give creditors greater incentive to monitor and Gârleanu and Zwiebel (2009) show that they help to allocate decision rights efficiently given asymmetric information.

may add credibility to our idea that non-exclusive contracting is an important reason that interbank markets are heavily reliant on collateral.

Secured debt is super senior. The seniority of secured debt is a basic feature of US bankruptcy law. Here, we take this as given. However, this is not an ad hoc policy of the courts. Rather, it reflects important constraints on the ability to establish priority. In general, unsecured debts cannot easily be prioritized temporally, since contracts can be backdated. In contrast, secured debts can be prioritized temporally by physically transferring collateral or by publicly registering a security interest in a property registry. Thus, in the context of Ayotte and Bolton (2011), collateralization "reduce[s] uncertainty and discovery costs of third parties who seek to acquire rights in the same property," i.e. rights in the same collateral (p. 3403).

State-contingent collateralization. We have assumed that state-contingent collateralization is impossible. But, if it were possible, it could circumvent the inefficiencies arising in our analysis. At Date 0, B would commit to collateralize Project 0 to C_0 in state L but not in state H, thereby allowing B to take on new debt in state H, where it is efficient, but not in state L, where it is not. However, as in the baseline model, B would prefer to collateralize Project 0 to C_1 in state L, reneging on his promise to collateralize it to C_0.[21] Thus, contingent collateralization effectively requires the commitment not to collateralize in the future, which we have argued can be impossible. Furthermore, even bilateral contingent contracting can be difficult in reality for a number of reasons established in the literature.[22] Indeed, "complete" contingent contracts are rare in practice.

Moreover, collateralization often requires a physical transfer of assets between the borrower and the creditor; in legal parlance, the secured debt is "possessory." In this case, state-contingent collateralization would require C_0 to be physically present at Date 1 to transfer possession, which could be costly or infeasible. This provides an additional rationale for our assumption that collateralization cannot be made state-contingent.[23]

[21] Analogously, C_0 could take extra collateral at Date 0 and commit to return (some of) it to B in state H. However, C_0 would prefer to renege and keep the collateral.

[22] These include the absence of common knowledge of the state (Aghion et al. (2012)), limited cognition (Bolton and Faure-Grimaud (2009), Tirole (2009)), fairness concerns (Hart and Moore (2008)), and the inability to commit not to renegotiate in complex environments (Hart and Moore (1999), Segal (1999)).

[23] If C_0 is present at Date 1, then there is another way to achieve efficiency: lend secured with interest and commit to transfer I_1 to B in state H. This resembles a loan commitment, an important part of borrowers' capital structure. Further, the arrangement does not require a transfer of collateral at Date 1, but only a transfer of cash. However, it may still be hard to implement sometimes. It requires that contracts specify state-contingent transfers, in addition to C_0 being there to make them. Moreover, C_0 must be able to commit to make transfers it would prefer to renege on ex post (even without posting collateral). We rule this out under the umbrella of state-contingent collateralization, since the proportion of the loan that is collateralized is different in state H (when there is under-collateralization) and state L (when there is over-collatearlization). Thanks to Martin Oehmke for pointing this out.

5 Conclusion

We have considered a model in which collateral serves to protect creditors against dilution with new secured debt. High pledgeability increases the risk of dilution, since it makes it easy to take on new secured debt and thus, paradoxically, makes creditors less willing to lend unsecured. Collateralization is required to protect against future collateralization—there is a collateral rat race.

This reliance on collateral leads to a collateral-overhang problem, whereby collateralized assets are encumbered and cannot be used to raise liquidity. We find that increasing the supply of collateral can aggravate this problem, by triggering the collateral rat race. Likewise, so can upholding the absolute priority rule, by which secured creditors get paid first in bankruptcy.

Appendix

Proof of Lemma 1

To see that B always wants to borrow from C_0, observe that he gets zero if he does not. This is because if B does not invest in Project 0 he cannot invest in Project 1 either, since

$$\theta X_1^s < I_1^s \tag{12}$$

for $s \in \{L, H\}$, by the combination of Assumption 1 and Assumption 3.

To see that B always wants to borrow from C_1, suppose he has debt with face value F_0 to C_0. It must be that $F_0 \geq I_0$ by C_0's participation constraint. Thus, if B does not borrow from C_1, he gets at most $X_0 - I_0$. Now, if B borrows from C_1 in state s, he gets at least his default payoff of $(1-\theta)(X_0 + X_1^s)$. Thus, a sufficient condition for B to borrow is that

$$(1-\theta)\big(X_0 + X_1^s\big) \geq X_0 - I_0 \tag{13}$$

which reduces to $(1-\theta)X_1^s \geq \theta X_0 - I_0$, which is implied by Assumption 4. $\square$

Proof of Lemma 2

Suppose B does not borrow in state L, so his pledgeable cash flow is θX_0 in state L. Thus, the expected cash flow that B can pledge to C_0 is at least $\mathbb{P}[H] \times 0 + \mathbb{P}[L] \times \theta X_0 = (1-p)\theta X_0$. This is greater than I_0 by Assumption 1. Thus, B can pledge enough to C_0 to satisfy its participation constraint. $\square$

Proof of Lemma 3

The result follows from Assumption 1 and Assumption 2.

Proof of Proposition 1

We first show that if $\theta < \theta^*$, B dilutes C_0 if and only if $X_1 = X_1^H$. Suppose C_0 lends unsecured, so B can make C_1 senior by borrowing secured. Hence, C_1 lends if and only if

$$\theta\big(X_0 + X_1^s\big) \geq I_1^s \tag{14}$$

or

$$\theta \geq \frac{I_1^s}{X_0 + X_1^s}. \tag{15}$$

This is always satisfied for $s = H$ by Assumption 5, but it is not satisfied for $s = L$ if $\theta < \theta^*$. Thus, there is no dilution in state L. Hence, C_0 lends by Lemma 2.

Now, if $\theta \geq \theta^*$, inequality (15) is satisfied for $s = H$ by Assumption 5 and is also satisfied for $s = L$. Thus, B borrows from C_1 in both states $s \in \{L, H\}$, and there is always dilution. Thus, C_0 does not lend unsecured, since, by Assumption 3, if B undertakes both projects, there is never enough pledgeable cash flow to repay both creditors. □

Contingent debt? Observe that the argument above for $\theta \geq \theta^*$ does not depend on our assumption that the repayments F_t do not depend on the state s. To see why contingent contracting cannot help, observe that in the first best C_0 must receive at least I_0 in state L, since it must get less than I_0 in state H. This is because the pledgeable payoffs are less than the project costs by Assumption 3 and C_1 must be guaranteed I_1^H in order to lend in state H (which is required in the first best). Thus, B must guarantee C_0 at least I_0 in state L. However, the argument above implies that, for high θ, B can never credibly promise I_0 to C_0 in state L, because he always dilutes C_0 with secured debt to C_1. □

Proof of Proposition 2

We first show that if B collateralizes a fraction σ_0 to C_0, B dilutes C_0 if and only if $X_1 = X_1^H$. This implies that B promises C_0 more than the cost I_0.

Suppose C_0 collateralizes the fraction σ_0 of X_0 to C_0, so B can make C_1 senior on the fraction $(1 - \sigma_0)$ of Project 0 and all of Project 1. Hence, C_1 lends if and only if

$$\theta\big((1 - \sigma_0)X_0 + X_1^s\big) \geq I_1^s. \tag{16}$$

The first best is attained if and only if this inequality is satisfied in state H and not in state L or

$$\frac{I_1^H - \theta X_1^H}{\theta X_0} \leq 1 - \sigma_0 < \frac{I_1^L - \theta X_1^L}{\theta X_0}. \tag{17}$$

Note that the left-most term is always less than one by Assumption 5 and the right-most term is always greater than zero by equation (12). Thus, there exists $\sigma_0 \in [0, 1]$ satisfying the condition as long as the left-most term is less than the right-most term, which holds as long as $I_1^H < I_1^*$ as in the hypothesis of the proposition. □

Proof of Proposition 3

From the proof of Proposition 2 (equation (17)), we know that the first best is attained only if

$$\frac{I_1^H - \theta X_1^H}{\theta X_0} \leq 1 - \sigma_0 < \frac{I_1^L - \theta X_1^L}{\theta X_0}. \tag{18}$$

In order for there to exist σ_0 satisfying this condition it must be that the left-most term is less than the right-most term. But that reduces to the violation of the hypothesis of the proposition that $I_1^H > I_1^*$. Thus, there is no σ_0 that implements first best.

To borrow from C_0, B must commit not to dilute in state L. But, by the argument above, B dilutes C_0 in state L whenever he does in state H for $I_1^H > I_1^*$. Thus, B must set σ_0 so high that he can never borrow from C_1. (Recall that B always prefers to borrow from C_0 than not to, by Lemma 1.) $\square$

Proof of Corollary 1

Since the equilibrium is efficient for $I_1^H \leq I_1^*$ (Proposition 2), we need to check for renegotiation proofness only when $I_1^H > I_1^*$, i.e. when the collateral overhang leads to inefficient underinvestment (Proposition 3). In this case, B must secure a large enough fraction σ_0 of Project 0 to C_0 that B cannot borrow from C_1 in state L, or

$$\theta\big((1-\sigma_0)X_0 + X_1^L\big) < I_1^L. \tag{19}$$

In order for renegotiation to be feasible, B, C_0, and C_1 must all be weakly better off. Thus, the combined payoff of C_0 and C_1 must weakly increase after renegotiation. Since they must invest I_1^H at Date 1, this implies the total repayment minus I_1^H must exceed the total payoff to C_0 absent renegotiation, or

$$\text{total repayment} - I_1^H \geq \sigma_0 \theta X_0. \tag{20}$$

Now, the limited pledgeability friction implies that the total repayment is at most $\theta(X_0 + X_1^H)$. Thus, for renegotiation to be feasible it must be that

$$\theta(X_0 + X_1^H) - I_1^H \geq \sigma_0 \theta X_0. \tag{21}$$

However, this cannot be satisfied together with equation (19) for $I_1^H > I_1^*$. Thus renegotiation is infeasible in this case. $\square$

Short-term debt. In the baseline model we assume that B cannot borrow from C_0 via one-period debt and roll over. We now show that this is without loss of generality if we restrict attention to renegotiation-proof debt.[24]

To consider short-term debt, we need to specify the sequence of moves at Date 1 and what happens if B defaults at Date 1. We assume that short-term debt matures after B has had the opportunity to borrow from C_1 and invest in Project 1, without loss of generality.[25] And we assume that C_0 gets the right to liquidate B's projects, but that their liquidation value is zero. Alternatively, B and C_0 can renegotiate, rescheduling the debt.

PROPOSITION 6. *Renegotiation-proof short-term debt does not improve on the implementation of long-term contracts.*

Proof. The result follows immediately from the fact that B has no cash flows at Date 1, so C_0 has zero recovery value in the event of liquidation. As a result, C_0 always prefers to accept a rescheduling to Date 2 than to liquidate at Date 1 and hence B has incentive to dilute C_0's unsecured debt, even if it is short term.

(Note also that short-term *secured* debt leads to exactly the same collateral overhang as long-term secured debt: it prevents B from borrowing from C_1 in state H when dilution is efficient.) □

Proof of Corollary 2

The result follows from Proposition 3 given the expression for I_1^* (equation 4). □

Proof of Corollary 3

The equilibrium outcomes follow from Proposition 1, Proposition 2, and Proposition 3. The face value F_0 when B invests in state H and not in state L (i.e. when $\theta < \theta^*$ or $I_1^H < I_1^*$) follows from C_0's break-even condition. C_0 is diluted in state H, since C_1 is senior on the secured debt with face value I_1^H. Hence, C_0 is repaid $\theta(X_0 + X_1^H - I_1^H)$ with probability p and F_0 with probability $1 - p$. Its break-even condition is

$$I_0 = p\theta(X_0 + X_1^H - I_1^H) + (1 - p)F_0. \tag{22}$$

Solving for F_0 gives the expression in the corollary. □

[24] Note that if we do not require renegotiation-proofness, then short-term debt, combined with state-contingent repayments, can indeed help. It effectively plays the role of state-contingent collateralization (as discussed in Section 4): B could borrow from C_0 in exchange for repayments at Date 1 in state L and repayments at Date 2 in state H. If C_0 *commits* to liquidate when B defaults on its short-term debt, then B will not dilute in state L to avoid liquidation. (But he will still dilute efficiently in state H.)

[25] This is without loss of generality because if, on the contrary, the debt matured earlier, then B could not repay it since his projects do not payoff until Date 2.

Proof of Lemma 4

Since B always borrows from C_1 if he can (Lemma 1), B borrows when he is unconstrained at Date 1, or whenever

$$\mu_1 \theta X_1^s + \frac{1}{2}\Big((1-\sigma_0)\theta X_0 + (1-\mu_1)\theta X_1^s\Big) \geq I_1^s. \tag{23}$$

To understand the expression on the left, recall that C_1 is senior on the collateralized portion of Project 1, which is at most $\mu_1 \theta X_1^s$, and that C_0 and C_1 are on equal footing for the uncollateralized portion of each project, i.e. C_0 and C_1 Nash bargain over $(1-\sigma_0)\theta X_0$ and $(1-\mu_1)\theta X_1^s$. The inequality reduces immediately to the condition in the proposition (equation (7)). □

Proof of Proposition 4

B borrows from C_1 in state L whenever

$$\mu_1 \theta X_1^L + \frac{1}{2}\Big((1-\mu_0)\theta X_0 + (1-\mu_1)\theta X_1^L\Big) \geq I_1^L, \tag{24}$$

where the expression on the left is determined as in the proof of Lemma 4 with the maximum amount of collateralization of Project 0, i.e. $\sigma_0 = \mu_0$. The inequality reduces to $\mu_1 \geq \mu_1^*$, the condition in the proposition. Thus, C_0 gets repaid less than I_0 in state L by Assumption 3. This implies that C_0 does not lend if state L is sufficiently likely, i.e. if p is not too large, as assumed in the proposition. (B cannot borrow at Date 1 either by equation (12).) □

Proof of Corollary 4

Immediately from the definition of σ_0^* in equation (9), we have that

$$\sigma_0^* = 1 - \frac{2I_1^L - \theta(1+\mu_1)X_1^L}{\theta X_0}, \tag{25}$$

which is increasing in μ_1. □

Proof of Proposition 5

B can finance Project 0 only if his pledgeable cash flow exceeds I_0. B can borrow from C_0 via unsecured debt if (i) Project 0's unsecured pledgeable cash flows are sufficient to cover the investment and (ii) C_0 is not at risk of dilution by the new debt to C_1 in state L. Condition

(i) says that

$$\theta_u X_0 \geq I_0 \tag{26}$$

and condition (ii) says that

$$\theta_c(X_0 + X_1^L) < I_1^L. \tag{27}$$

These conditions hold together if and only if $\theta \in [\theta_u^*, \theta_c^*)$, as required in the proposition. Thus, B borrows from C_0 and invests at Date 0 and does not borrow from C_1 and does not invest at Date 1 in state L. Assumption 5 implies that B does borrow from C_0 and does invest at Date 1 in state H. Thus, the first best is attained. $\square$

Proof of Corollary 5

The result follows from the same argument as Proposition 3. Specifically, set $\theta = \theta_c$ in equation (18) above. $\square$

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